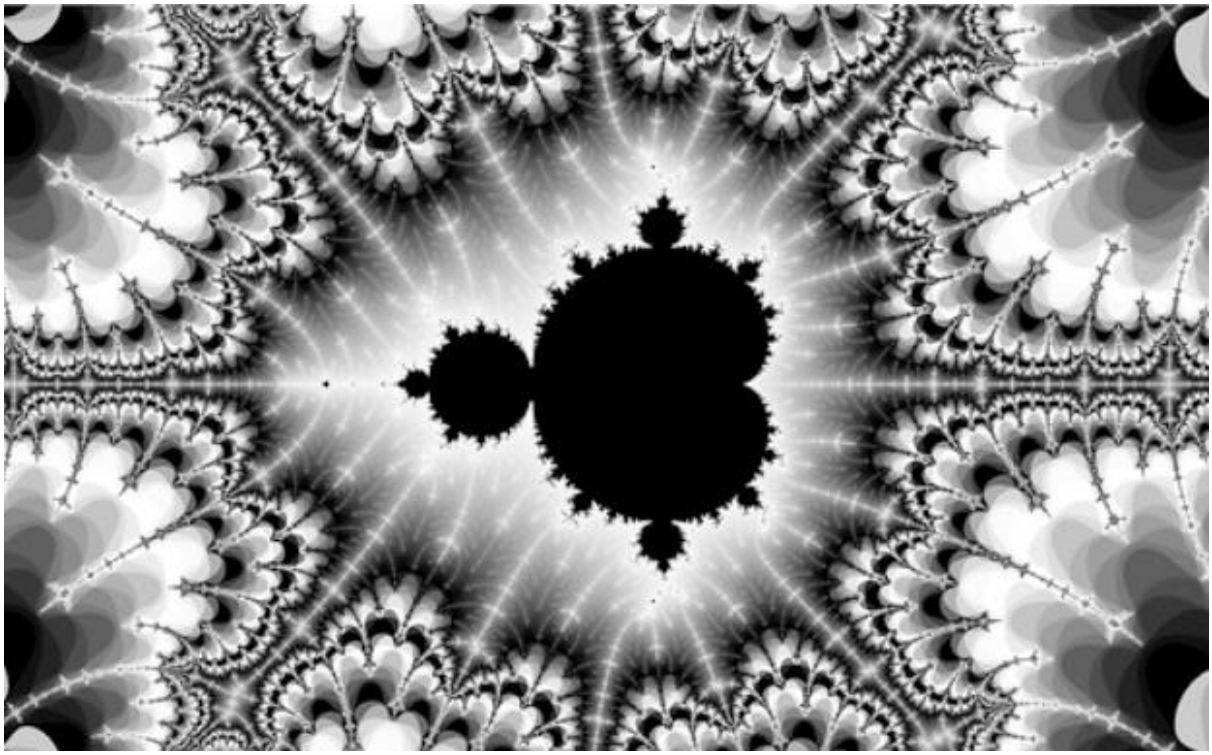
Plymstock School

Achieving Excellence through Curriculum and Culture

A Level Mathematics Summer Work



Task one

You need to be confident at answering the following questions.

Make sure you show your workings out and correct notation for each question.

To help, you can use Mathswatch (your logins will still work until the end of the summer).

Types of Number and Fractions

1. Which of the following are integers?

$$4 \qquad -3.5 \qquad 0.3 \qquad \frac{4}{5} \qquad 8.99 \qquad -10 \qquad 205 \qquad 0$$

2. Which of the following values are rational, and which are irrational?

$$5.\dot{9} \qquad \pi \qquad \sqrt{7} \qquad \frac{1}{5} \qquad -6 \qquad \sqrt{4} \qquad 13.978 \qquad 2.1$$

3. Evaluate the following without using a calculator, giving your answers in their lowest terms. Give any answers larger than 1 as improper fractions.

a) $\frac{2}{9} \times \frac{3}{5}$

b) $\frac{1}{6} \div \frac{2}{3}$

c) $\frac{1}{12} + \frac{5}{6}$

d) $\frac{8}{5} - \frac{1}{7}$

Indices, Multiplying Out Brackets and Factorising

4. Simplify the following:

a) $x^7 \times x^2$

b) $10y^3 \div 5y$

c) m^0

d) $(2n^3)^5$

5. Write 5^{-2} as a fraction.

6. Evaluate the following without using a calculator:

a) $\left(\frac{3}{4}\right)^2$

b) $16^{\frac{1}{2}}$

c) $8^{\frac{2}{3}}$

d) $36^{-\frac{1}{2}}$

7. Multiply out the brackets and simplify your answers where possible.
   a) $(x+4)(x-6)$
   b) $(x+5)^2$
   c) $(2x-1)(x+3)$
   d) $(x+1)(x-4)(x+5)$

8. Factorise the following:
   a) $5x+20$
   b) $3a+12b$
   c) x^2-4
   d) $9x^2-36$
   e) x^2-5

Surds

9. Simplify the following:
   a) $\sqrt{3}\times\sqrt{2}$
   b) $\left(\sqrt{5}\right)^2$
   c) $\frac{\sqrt{30}}{\sqrt{5}}$
   d) $\sqrt{12}+2\sqrt{3}$
   e) $\left(1+\sqrt{7}\right)^2$

10. Rationalise the denominators of the following:
   a) $\frac{3}{\sqrt{2}}$
   b) $\frac{\sqrt{5}}{2\sqrt{2}}$
   c) $\frac{2}{3+\sqrt{6}}$
   d) $\frac{\sqrt{2}}{1-\sqrt{5}}$

Solving Equations and Rearranging Formulae

11. Solve the following:
   a) $5x-2=8$
   b) $3(x-6)=2(x-4)$
   c) $\frac{x+2}{3}+\frac{2x}{5}=x+2$
   d) $2x(x+1)=2x+18$

12. Make x the subject of the following formulae:
   a) $y=mx+c$
   b) $y=\frac{3x+2}{5}$
   c) $y=2x^2z+1$
   d) $y=\frac{3x+1}{x-2}$

Quadratic Equations

13. Solve the following by factorising:

a) $x^2 - 3x + 2 = 0$

b) $x^2 + 6x + 5 = 0$

c) $2x^2 - 3x - 5 = 0$

d) $3x^2 - 13x = -12$

14. Solve the following using the quadratic formula. Give your answers to two decimal places.

a) $x^2 + 2x - 10 = 0$

b) $2x^2 - 5x - 1 = 0$

15. Solve the following by completing the square. Give your answers as surds.

a) $x^2 - 4x - 2 = 0$

b) $2x^2 + 4x - 7 = 0$

16. Complete the square for $x^2 + 6x + 8$. Hence sketch the graph of $y = x^2 + 6x + 8$, labelling the turning point and intercepts with the x-axis.

Algebraic Fractions, Inequalities and Simultaneous Equations

17. Simplify the following:

a) $\frac{15a^3b^3}{5a^2b}$

b) $\frac{2x^2y}{(4xy)^2}$

c) $\frac{x^2-16}{x^2-x-20}$

18. Simplify the following:

a) $\frac{9b^2}{a} \times \frac{2a^2}{3b}$

b) $\frac{2(x-1)^2}{15} \times \frac{10}{4x-4}$

c) $\frac{3x^2-21x}{x+2} \div \frac{x(x+7)}{9x+18}$

d) $\frac{3}{x+1} + \frac{2x-3}{x^2}$

19. Solve the following inequalities:

a) $7x + 5 \leq 2x$

b) $2(10 - x) > 4$

c) $2x^2 + 3 < 21$

d) $4x^2 - 9 \geq 7$

e) $x^2 - 4x + 10 \geq 2x + 5$

20. Draw a set of axes with the x-axis from -2 to 3 and the y-axis from 0 to 6.
Show on these axes the region that satisfies the following inequalities:
$y > 3x - 1$, $y < x + 3$ and $y \geq \frac{x}{5} + 2$

21. Solve the following simultaneous equations:
    a) $2x + y = 2$
       $x - 3y = 8$

    b) $3x - 2y = 1$
       $5x - 3y = 7$

    c) $y = x^2 + 3$
       $y - 2x = 18$

    d) $3y = 2(x^2 - 3)$
       $2x - y = 2$

Proof and Functions

22. Prove that the sum of any three consecutive odd numbers is a multiple of 3.

23. Mia says, "for any integers x and y, $xy > y$". Prove that Mia is wrong.

24. $f(x) = \frac{x+5}{3}$ and $g(x) = x - 3$.
    a) Evaluate $f(4)$
    b) Find $fg(x)$
    c) Find $f^{-1}(x)$

Straight Lines and Quadratic Graphs

25. Give the gradient and y-intercept of the line $x + 2y = 4$

26. Point A has coordinates (5,2) and point B has coordinates (2,-4).
    a) Find the equation of the line passing through points A and B.
    b) Find the exact length of line AB.

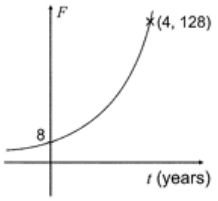
27. Line A has equation $y = 2x + 5$.
    a) Find the equation of the line parallel to line A which passes through (3,2).
    b) Find the equation of the line perpendicular to line A which passes through (2,1).

28. Sketch the graph of $y = x^2 - 8x + 15$. Label the graph with the coordinates of the turning point and the points where the graph crosses the axes.

Harder graphs and Graph Transformations

29. Sketch the following graphs:
    a) $y = x^3$
    b) $y = \frac{1}{x}$
    c) $y = -\frac{1}{x}$

30. The graph on the right shows how the number of fish (F) living in a river changes over time. The equation of the graph is $F = mn^t$ where t is the number of years and m and n are positive constants. Find the values of m and n.

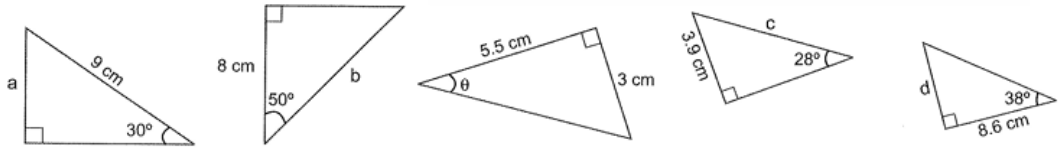


31. Find the equation of the tangent to $x^2 + y^2 = 25$ at the point (3,4). Give your answer in the form $ax + by + c = 0$.

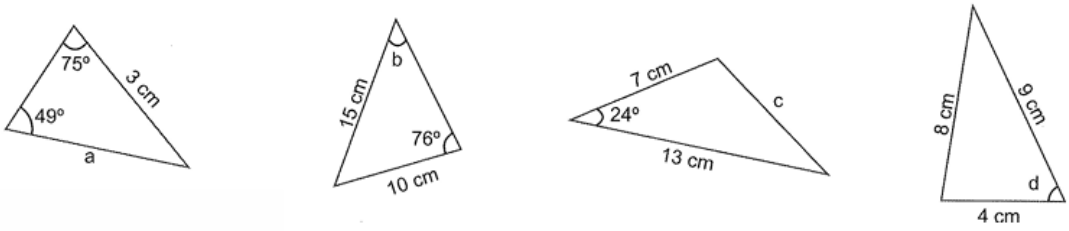
32. $f(x) = x^2$. For parts a) to c) below, sketch the graphs of $y = f(x)$ and the given transformation.
    a) $y = f(x) + 3$
    b) $y = f(x + 3)$
    c) $y = -f(x)$

Trigonometry and Vectors

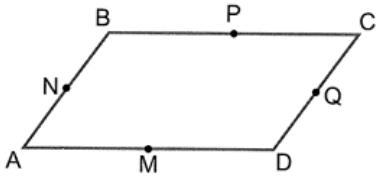
33. Find the unknowns in each of these triangles. Give your answers to 1 decimal place.



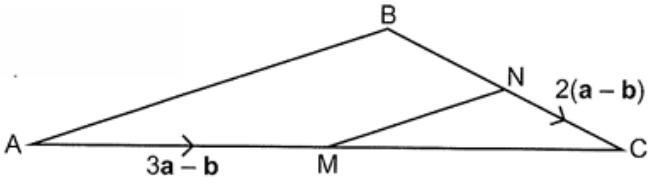
34. Find the unknowns in each of these triangles. Give your answers to 1 decimal place.



35. $ABCD$ is the parallelogram shown on the right. M, N, P and Q are the midpoints of the sides. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.
    Find the following vectors in terms of **a** and **b** .
    a) $\overrightarrow{AC}$
    b) $\overrightarrow{DQ}$
    c) $\overrightarrow{CM}$
    d) $\overrightarrow{QP}$
    e) $\overrightarrow{MB}$
    f) $\overrightarrow{PA}$



36. The diagram shows triangle ABC. M is the midpoint of $\overrightarrow{AC}$ and N is the midpoint of $\overrightarrow{BC}$. $\overrightarrow{AM} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{NC} = 2(\mathbf{a} - \mathbf{b})$. Show that $\overrightarrow{AB}$ and $\overrightarrow{MN}$ are parallel.



Task Two

Equipment

For maths you will need:

- Paper
  You will need to bring your own paper for lessons. You will produce a lot more written work than at GCSE.
- Folders
  You need to keep your notes and work organised. You will do different topics with each of your teachers so notes will not follow on from each other lesson by lesson. You will need to be organised!
- Pens, pencils, etc.

You need to ensure that you have the equipment you need for your lessons.

Calculators

For the A level maths (& further maths) course you will need a more advanced calculator than your Casio GCSE calculator, although you can still use this for basic calculations. We recommend and use the Casio FX-CG50.

NOTE: You **DO NOT** need to go out and buy a brand new calculator before September. Here at school we offer to lend you one for the entirety of the A level course (a deposit will need to be paid that we return after you have returned the calculator in full working order with the lid) or you can order one through school (if you are planning on continuing with your mathematical studies beyond A level this is recommended); we get a school discount that means you can buy one much more cheaply.

More information will be given in September after you have started your A Level maths course.