

## Maths tutorial booklet for

### M0: Arithmetic and numerical computation

Name: \_\_\_\_\_

Target grade: \_\_\_\_\_

Quiz scores:

M0.1 Recognise and make use of appropriate units in calculations =

M0.2 Recognise and use expressions in decimal and standard form =

M0.3 Use ratios, fractions and percentages =

M0.4 Estimate results =

M0.5 Using calculators to find and use power, exponential and logarithmic functions =

## Maths skills – M0.1 Recognise and make use of appropriate units in calculations

### Tutorials

Learners may be tested on their ability to:

- convert between units e.g.  $\text{mm}^3$  to  $\text{cm}^3$  as part of volumetric calculations
- work out the unit for a rate e.g. breathing rate.

### Units

Units indicate what a given quantity is measured in. Generally, a measured quantity without units is meaningless, although there are some exceptions, like pH for example.

In an exam, you would be expected to be able to convert between different metric units, such as between mm and cm without being given the information about how many mm there are in a cm.

Converting between units involves multiplying or dividing by an appropriate factor. The factor is determined by how many of one unit there are in the other unit. For example, there are 1000 m in a km, so in this case, converting between metres and km your conversion factor would be 1000.

If your measurement is getting larger, for example from mm to m, then you need to divide by your factor. If your measurement is getting smaller, for example from m to mm then you need to multiply by your factor.

Just remember that if the unit gets bigger the number should get smaller, and if the unit gets smaller then the number should get bigger.

For example, say we are converting  $7 \mu\text{m}$  to mm. There are 1000  $\mu\text{m}$  in one mm, so the conversion factor is 1000 or  $10^3$ .

As the measurement is getting larger, you divide your number by the factor.

Therefore, we divide  $7 \mu\text{m}$  by  $10^3$  which equals 0.007 mm which in standard form (see section M0.2 for this) is  $7 \times 10^{-3}$  mm.

Let's look at units of length you may encounter and how they relate to the metre.

We have kilometres which are 1000 m, decimetres are a tenth of a metre, centimetres a hundredth, millimetres a thousandth of a metre, micrometres are  $10^{-6}$  metres and nanometres are  $10^{-9}$  metres.

Units of length	Abbreviation	Number of metres	
kilometre	km	1000 m	$10^3$ m
metre	m	m	m
decimetre	dm	1/10 m	$10^{-1}$ m
centimetre	cm	1/100 m	$10^{-2}$ m
millimetre	mm	1/1000 m	$10^{-3}$ m
micrometre	$\mu\text{m}$	1/1000000 m	$10^{-6}$ m
nanometre	nm	1/1000000000 m	$10^{-9}$ m

These units of length are also used when dealing with area and volume.

Area is expressed as length squared, such as square centimetres.

You may need to work out how many square centimetres there are in a square metre. There are 100 cm in a metre, so if you were to convert cm to metres the conversion factor would be 100. But you are dealing with squares, so there are 100 **times** 100  $\text{cm}^2$  in a  $\text{m}^2$ . So here your conversion factor would be 100 times 100, which is 10,000, or  $10^4$ .

So 4  $\text{m}^2$  expressed in square centimetres would be  $4 \times 10,000 \text{ cm}^2$ . Remember that here the units are getting smaller – metres to centimetres – so the number gets bigger – you multiply by your factor.

Likewise, volume is expressed as length cubed.

So 4  $\text{m}^3$  expressed in  $\text{cm}^3$  would have a conversion factor of  $100 \times 100 \times 100$ , which is  $10^6$ . So there are  $10^6 \text{ cm}^3$  in one  $\text{m}^3$  and 4  $\text{m}^3$  expressed in  $\text{cm}^3$  would be  $4 \times 10^6 \text{ cm}^3$ .

The main units of volume are  $\text{mm}^3$ ,  $\text{cm}^3$  and  $\text{dm}^3$ . However, if we are dealing with liquids we often use the unit 'litre' (l) instead. Here are the more commonly used units of volume for liquids. Note that a litre is equivalent to  $\text{dm}^3$  and a millilitre is equivalent to  $\text{cm}^3$ .

Units of volume	Abbreviation	How it relates to the litre		Equivalence in length measurements
litre	<i>l</i>	<i>l</i>	<i>l</i>	dm <sup>3</sup>
millilitre	<i>ml</i>	10 <sup>-3</sup> <i>l</i>	1/1000 <i>l</i>	cm <sup>3</sup>
microlitre	$\mu$ <i>l</i>	10 <sup>-6</sup> <i>l</i>	1/1000000 <i>l</i>	mm <sup>3</sup>
nanolitre	<i>nl</i>	10 <sup>-9</sup> <i>l</i>	1/1000000000 <i>l</i>	

You will encounter units of mass and here are common units of mass and how they relate to the gram. For example the microgram is a millionth of a gram.

Units of mass	Abbreviation	How it relates to the gram	
kilogram	kg	10 <sup>3</sup> g	1000 g
gram	g	g	g
milligram	mg	10 <sup>-3</sup> g	1/1000 g
microgram	$\mu$ g	10 <sup>-6</sup> g	1/1000000 g
nanogram	ng	10 <sup>-9</sup> g	1/1000000000 g

And here are units of time and how they relate to the standard unit of the second.

Units of time	Abbreviation	How they relate to the second	
second	s	s	s
millisecond	ms	10 <sup>-3</sup> s	1/1000 s
microsecond	$\mu$ s	10 <sup>-6</sup> s	1/1000000 s
nanosecond	ns	10 <sup>-9</sup> s	1/1000000000 s

If we are dealing with times greater than a second, we use minutes, hours, days, etc.

You must always think about which unit you are using when communicating a measurement. For example, although it is true that an average mitochondrion has a length of 0.0000055 m, it is more appropriate to state the measurement as 5.5  $\mu\text{m}$ . Likewise, if the volume of a liquid is 56000 m<sup>3</sup> it would be better to refer to it as 56 l or 56 dm<sup>3</sup>.

It is the clear communication of the number and its unit which is the most important thing to consider.

## Rates of change

A rate of change is the quantity being measured per unit of time. Now this **per** means 'divided by'. So quantity measured divided by the unit of time equals the rate of change.

An important mathematical notation to remember is that whatever is underneath the division line (the denominator) can also be written to the negative power. So the rate of change would be equal to the quantity being measured times the unit of time to the minus one.

So remember, whenever you see the term 'per' it means 'divided by' and you need to write the units with the correct mathematical notation, with the denominator expressed to its negative power.

For example, a woodlouse might crawl at 10 cm per second. This would be written as 10 cm s<sup>-1</sup>.

If a patient is on a drip, the number of drips per minute would be drips min<sup>-1</sup>.

There are other examples where the rate of change is measured as quantity per length, area or volume. Here the same principle applies, with, for example, the unit volume being made into its negative power:

For example, as a slime mould develops the number of cells per unit volume might increase, so you would be looking at the number of cells per mm<sup>3</sup> or cells mm<sup>-3</sup>. The power stays the same, but the negative sign in front of it tells you that you divide by mm<sup>3</sup>. So the rate of change of the slime mould colony would be measured in number of cells per cubic mm per s, written as cells mm<sup>-3</sup> s<sup>-1</sup>.

There are other examples where you would need to combine two or more units, for example, light energy is measured in photons per square metre per second – the number of photons that hit a square metre every second and you would express your data in photons m<sup>-2</sup> s<sup>-1</sup>. The rate of change would be measured in a change in the number of these photons hitting a

square metre every second over a period of time. So if you were to measure the rate of *change* in light intensity you would express your data in photons per square metre per second per second: photons  $\text{m}^{-2} \text{s}^{-1} \text{s}^{-1}$  which is written as photons  $\text{m}^{-2} \text{s}^{-2}$

Rates of change are used in many areas of biology: For example:

Bacterial growth rates are measured as the number of bacteria per hour expressed as bacteria  $\text{h}^{-1}$

Breathing rate would be expressed as breaths  $\text{min}^{-1}$

Rate of change in temperature would be  $^{\circ}\text{C s}^{-1}$ .

You just need to remember that the minus sign is simply a notation that tells you the unit is the denominator.

So if we return to the example of light energy, measured in photons  $\text{m}^{-2} \text{s}^{-2}$ , this tells you that you take the change in the number of photons hitting each square metre each second and divide that by the time over which the change took place.

## Maths skills – M0.1 Recognise and make use of appropriate units in calculations

### Quiz – converting between units

Answers to all of these questions should use standard form e.g. use  $5.6 \times 10^3$  rather than 5600, use  $4.2 \times 10^{-2}$  rather than 0.042.

1. How many?

mm in a m


$\mu\text{m}$  in a m

nm in a mm

mm in a  $\mu\text{m}$

$\mu\text{m}$  in a nm

$\mu\text{l}$  in a litre

$\mu\text{m}$  in a mm

nm in a  $\mu\text{m}$

nm in a m

m in a  $\mu\text{m}$

mm in a nm

$\text{m}^3$  in a litre


mm in a m

--

$\mu\text{l}$  in a  $\text{m}^3$

--

$\mu\text{s}$  in a ms

--

$\mu\text{m}$  in a mm

--

ms in a s

--

2. Convert each of the following into metres.

(a) 70 nm

--

(b) 5  $\mu\text{m}$

--

(c) 1 mm

--

(d) 0.2 mm

--

3. Convert each of the following into  $\mu\text{m}$ .

(a) 4 m

--

(b) 200 nm

--

(c) 17 mm

--

(d) 0.3 nm

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4. Areas. How many?

(a)  $\mu\text{m}^2$  in a  $\text{m}^2$

--

(b)  $\mu\text{m}^2$  in a  $\text{mm}^2$

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5. Volumes. How many?

(a)  $\text{mm}^3$  in a  $\text{cm}^3$

(b)  $\mu\text{m}^3$  in a  $\text{mm}^3$


6. Convert each of these into more sensible units using standard form to express your answers if appropriate.

(a) 0.0003  $\mu\text{m}$

(b) 0.004 km

(c) 4500000 nm

(d) 0.0007 s




### Quiz – Rates of change

1. Express these rates of change with the correct units:

(a) 2  $\mu\text{g}$  per  $\text{cm}^3$

(b) 200 kJ per  $\text{m}^2$  per year

(c) 10 g per  $\text{dm}^3$

(d) 15  $\text{cm}^3$  per minute

2. In an experiment you were measuring the growth rate of *Salmonella*. You started with 100 *Salmonella* and after 2 hours you had 6500 *Salmonella*. What is the bacterial growth rate?

3. In an experiment you were measuring the growth rate of *Salmonella*. You started with 80 *Salmonella* and after 4 hours you had 5000 *Salmonella*. What is the bacterial growth rate?

4. How would you express the following in numbers and units?

a) A woodlouse crawled 5 cm in 10 min.

b) A patient's drip flowed with 10 drips every 30 s.

c) The growth of a slime mould colony was 40 cells per millimetre cubed per hour.

d) A breathing rate of 20 breaths in 30 s.

e) A change in temperature of 1.2 degrees over three years.

## Maths skills – M0.2 Recognise and use expressions in decimal and standard form

### Tutorials

Learners may be tested on their ability to:

- use an appropriate number of decimal places in calculations, e.g. for a mean
- carry out calculations using numbers in standard and ordinary form, e.g. use of magnification
- understand standard form when applied to areas such as size of organelles
- convert between numbers in standard and ordinary form
- understand that significant figures need retaining when making conversions between standard and ordinary form, e.g.  $0.0050 \text{ mol dm}^{-3}$  is equivalent to  $5.0 \times 10^{-3} \text{ mol dm}^{-3}$ .

### Decimal places

You need to understand and use the appropriate number of decimal places or significant figures in calculations.

Decimal places are simply how many digits come after the decimal point.

For example, all of these numbers have the same number of decimal places but may have a different number of significant figures.

Number	Decimal places	Significant figures
11.37	2	4
105.82	2	5
5.69	2	3
0.02	2	1

Significant figures (covered in detail in section M1.1) are numbers with meaning, starting from the first non-zero digit. For example.... now these numbers have the same number of significant figures but differ in the number of decimal places.

Number	Decimal places	Significant figures
11.3	1	3
105	0	3
5.69	2	3
0.0200	4	3

When you are carrying out an experiment, you will be expected to record raw data to the same number of decimal places (rather than the same number of significant figures). These decimal places are used to tell the reader the accuracy of your data collection. For example when measuring volumes 5.0, 5.5, 6.0 and 6.5 ml half a ml would be about right if you were using a 10 ml measuring cylinder. However, if you used a Gilson pipette you may be justified in using two decimal places.

Measurement	Volume (ml) X	Volume (ml) ✓
1	9	9.0
2	9.5	9.5
3	10	10.0
4	10.5	10.5

Processed data (i.e. what has been calculated), can be recorded up to one decimal place more than the raw data. For example, if you were asked to calculate the mean for the above example, from the data using a measuring cylinder the answer could be recorded as 9.75 ml. If you were using the data from the Gilson pipette, you could express the mean as 9.750 ml.

Measurement	Volume (ml) X	Volume (ml) ✓
1	9	9.0
2	9.5	9.5
3	10	10.0
4	10.5	10.5
		Mean = 9.75

When adding and subtracting decimals the answer should be given using the lowest number of decimal places used in the calculation. This is because you can't claim your degree of accuracy to be any higher than what your data say. This involves rounding off the number to a certain point depending on how many decimal places you are recording, with rounding up being required if the next number is 5 or above. For example,  $25.5 - 8.31 = 17.19$ . The lowest number of decimal places in the calculation is one decimal place so the answer would therefore be 17.2.

This is particularly important when measuring quantities by difference such as mass, temperature and volume. The measurements made should be recorded to a specific number of decimal places, and when calculating the difference between the measurements, this number of decimal places should be maintained.

For example, if the following temperature measurements were recorded:

Initial temperature                      22.5 °C

Final temperature                        29.5 °C

The temperature difference would be  $29.5 - 22.5 = 7.0$  °C. This is given to 1 decimal place, to match the resolution of the measured values as this is the degree of accuracy of our data. The '0' is significant, so must be included.

The number of decimal places you show in your collection of raw data or in your answer at the end of a calculation is communicating information about the degree of accuracy you are claiming, and this is important information in a scientific context.

## Standard Form

Many numbers in biology, particularly with calculations involving large or small numbers, will be written in a scientific notation known as standard form. A number written in standard form is:

$$a \times 10^n$$

where  $n$  is a whole number (also known as an integer)

and  $a$  is between the values of 1 and 10

Standard form makes writing and reading long numbers easier. For example, 1 million in standard form is  $1 \times 10^6$ .

2 million would be  $2 \times 10^6$ .

You need to remember the “power laws” in order to deal with the value of  $n$ .

The multiplicative rule is where you add the powers

The division rule is where you subtract one power from the other

The power rule is where the powers are multiplied

The reciprocal rule is where the negative sign tells you it is the denominator

And the root rule is where the power is divided by the root

For example, for  $2 \times 10^4 \times 8 \times 10^3$ , we know that  $2 \times 8 = 16$  so our  $a$  value is 16. For the  $n$  value we need to apply the multiplicative rule and add the powers, so  $10^4 \times 10^3$  equals  $10^7$ . Therefore the whole answer is  $16 \times 10^7$ .

However, be careful that you don't make a mistake by forgetting that  $a$  must be between 1 and 10.

So here the multiplicative power rule has been applied and there is the correct numerical answer, the answer is not in standard form because the  $a$  value is not between 1 and 10. Therefore, 16 needs to be ‘scaled’ down by dividing by 10. Importantly - if we divide the numerical answer by 10, we have to increase the power by 1 in order to keep the numerical value the same. Therefore  $16 \times 10^7$  becomes  $1.6 \times 10^8$ .

Here are some other examples of numbers written using standard form:

Number	Standard form
456000	$4.56 \times 10^5$
2068567	$2.068567 \times 10^6$
46	$4.6 \times 10^1$
3789	$3.789 \times 10^3$
4	$4 \times 10^0$
0.0005	$5 \times 10^{-4}$
0.4678	$4.678 \times 10^{-1}$
0.000030006	$3.0006 \times 10^{-5}$
0.3040506	$3.040506 \times 10^{-1}$

You are expected to be able to express results in standard form and to also convert between standard and decimal forms. For example, the size of a cell may be 0.0001 m in ordinary (decimal) form. This would be written in standard form as  $1 \times 10^{-4}$  m.

In biology raw data is usually reported to the same number of decimal places rather than the same number of significant figures including when converting between standard and decimal forms.

By applying this to the data from the previous table, we get this third column:

Number	Standard form	Standard form to 1 decimal place
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456000	$4.56 \times 10^5$	$4.6 \times 10^5$
2068567	$2.068567 \times 10^6$	$2.1 \times 10^6$
46	$4.6 \times 10^1$	$4.6 \times 10^1$
3789	$3.789 \times 10^3$	$3.8 \times 10^3$
4	$4 \times 10^0$	$4.0 \times 10^0$
0.0005	$5 \times 10^{-4}$	$5.0 \times 10^{-4}$
0.4678	$4.678 \times 10^{-1}$	$4.7 \times 10^{-1}$
0.000030006	$3.0006 \times 10^{-5}$	$3.0 \times 10^{-5}$
0.3040506	$3.040506 \times 10^{-1}$	$3.0 \times 10^{-1}$

You have to be careful though because, by reducing the number of significant figures, you are altering your communication about the degree of accuracy you are claiming to have.

In M0.1 we looked at converting between units to express a measurement clearly. An alternative is to keep units the same but use standard form. This is most useful in biology when you want to make a clear comparison between two things. You might want to show that a bacterium was 2  $\mu\text{m}$  long and that an *Amoeba* was 1 mm. It would be better to communicate this as the bacterium being  $2 \times 10^{-3}$  mm and the *Amoeba* being 1 mm. Or you could say that the bacterium is 2  $\mu\text{m}$  and the *Amoeba* is  $1 \times 10^3 \mu\text{m}$ . In this way you are using standard form to communicate clearly the difference in length.



## Maths skills – M0.2 Recognise and use expressions in decimal and standard form

### Quiz

1. Convert each of these into standard form, to one decimal place.

1) 40000000

2) 8567

3) 0.0000007

4) 0.07607

5) 4500067

6) 93

7) 7

8) 62545


2. Multiply these numbers, giving your answer in standard form to one decimal place:

1)  $(1 \times 10^4) \times (6 \times 10^3)$

2)  $(3 \times 10^4) \times (3 \times 10^{-1})$

3)  $(2 \times 10^{-2}) \times (3 \times 10^{-3})$

4)  $(4 \times 10^2) \times (5 \times 10^3)$


## Maths skills – M0.3 Use ratios, fractions and percentages

### Tutorials

Learners may be tested on their ability to:

- Calculate percentage yields
- Calculate surface area to volume ratio
- Use scales for measuring
- Represent phenotypic ratios (monohybrid and dihybrid crosses)

### Percentage

'Percentage' means 'out of 100'

x percent means x divided by 100.

$$x\% = \frac{x}{100}$$

So a percentage is a fraction of 100. For example, 30 percent as a fraction is 30 out of 100

$$30\% = \frac{30}{100}$$

Let's look at how you convert 30/100 into a percentage. First you work out 30/100 as a decimal, which is 0.3, and then you multiply this by 100. Remember that percent means 'out of 100'.

Let's convert 30/110 into a percentage. Firstly work out 30/110 as a decimal – you might need a calculator to do this. And then multiply your answer by 100, which gives you 27%. Working backwards, 27% means 27 out of 100. And this is equivalent, or equal to, 30/110.

In biology you may be required to find a percentage of a quantity. For example in studying populations you might want to know the gender distribution in a population. To work out the percentage you need to divide the 'part' (the specific data you are looking at) by the 'whole' (the total number). To convert this into a percentage you multiply your answer by 100%.

$$\frac{\text{part}}{\text{whole}} \times 100\% = \%$$

For example, in a population of 2200 rabbits there are 1000 males and 1200 females. You want to know the percentage that is female. Here 1200 is the 'part' and 2200 is the 'whole'. So we divide 1200 by 2200, and multiply by 100% to express the answer as a percentage. The answer is 54.5%.

$$\% \text{ female} \rightarrow \frac{1200}{2200} \times 100\% = 54.5\%$$

Now we are going to look at percentage change

Percentage change is a useful calculation to compare values and determine how much a quantity has increased or decreased by.

In percentage change the 'part' is the increase and the 'whole' is the original quantity.

For example, you might subtract the initial mass from the final mass to find the increase. Then divide this by the initial mass. This is then multiplied by 100 to give the answer as a percentage. Doing it this way, you have to keep an eye on whether it is an increase or a decrease and make it clear in your answer.

For example, if the initial mass of some cellular tissue in an osmosis investigation is 5.6 g and after a day it is 4.7 g. You might want to find out the percentage change.

The percentage change is  $4.7 - 5.6$  divided by 5.6 and then multiplied by 100% which gives you a percentage change of -16%. You could refer to this as a 16% decrease.

$$\frac{4.7 - 5.6}{5.6} = -0.16$$

$$0.16 \times 100 = -16\% = 16\% \text{ decrease}$$

Another way to calculate percentage change uses the idea of a multiplier.

Percentage change is calculated using this formula:

The original quantity times a multiplier equals the new quantity

The key point to remember is that a multiplier of 1 represents a change of 0%, because if you substitute 1 for the multiplier in the equation, then the Original quantity equals the new quantity

The difference between the multiplier and 1, multiplied by 100%, is the % change.

So a multiplier of 1.43 represents an increase of 43 % because 1.43 minus one, multiplied by 100 % equals 43 %, whilst a multiplier of 0.83 represents a decrease of 17 % because 1 minus 0.83 = 0.17, multiplied by 100 % equals 17 %.

Let's return to the previous example where the initial mass of some cellular tissue in an osmosis investigation is 5.6 g and after a day it is 4.7 g

Then to work out the percentage change we have to work out the value of the multiplier:

The initial mass is 5.6 g, the final mass is 4.7 g.

Remember that the original quantity times the multiplier equals the new quantity.

Rearranging this equation to calculate the multiplier, gives you the new quantity divided by the original quantity, which is 4.7 divided by 5.6, which is 0.84.

Initial mass = 5.6 g, Final mass = 4.7 g

Original quantity x Multiplier = New Quantity

$$5.6 \times \text{Multiplier} = 4.7$$

$$\text{Multiplier} = \frac{4.7}{5.6} = 0.84$$

This represents a decrease of 1 - 0.84 which equals 0.16. To express this as a percentage you need to multiply by 100 % which gives you a 16 % decrease.

It doesn't matter which method you use as long as you understand the principles behind the method you adopt.

## Percentage yield

The percentage yield is a way of looking at the limitations of a process.

You can work out mathematically what goes in and what comes out, but there are many steps in a biological process and the actual yield is rarely the same as the theoretical yield.

The percentage yield is a measure of the limitations of a biological or chemical process. You are essentially saying what percentage of the theoretical yield am I ending up with at the end of this process?

One area where percentage yield is relevant is in scientific experiments where yield can be limited by the amount of a reagent that is present. In these situations it is important to be able to work out how efficient a reaction is under certain conditions. This is where it helps to calculate a percentage yield.

For percentage yield calculations, the actual yield – the amount of product actually obtained in a chemical reaction – is divided by the theoretical yield – the amount of product that could possibly be produced according to the starting amount of the limiting reagent. To convert this into a percentage you then multiply by 100 %.

For example, imagine a scenario where you were producing insulin from genetically modified bacteria. You calculated that with everything you started with at the beginning you had a theoretical yield of 120 mg of insulin. However, when you carried out the procedure you only ended up with 90 mg – the actual yield.

The percentage yield is the actual yield, 90 mg, divided by the theoretical yield, 120 mg, multiplied by 100 %, which equals 75 %

In a big manufacturing process these calculations are very important as getting the highest possible percentage yield might be the difference between a process being financially viable or not.

## Ratio

A ratio is a relationship between two numbers and is used to compare values, with the quantities being in direct proportion.

When presenting the ratio of one quantity to another it is reported in the form  $x : 1$ ,

$x$  is found by dividing the first quantity by the second. So, expressing 40 to 5 as a ratio, 40 would be divided by 5. This would be expressed as a ratio of 8 to 1.

$$x = \frac{40}{5} = 8 \quad \text{ratio} = 8:1$$

Surface area to volume ratio is important in understanding processes such as gas exchange, heat loss and diffusion rates. Surface area to volume ratio is affected by size and shape. So keeping the volume of an object the same, but changing its shape, changes its surface area to volume ratio. Keeping the shape the same, such as a sphere, but changing the volume will also affect its surface area to volume ratio.

The surface area to volume ratio is calculated by dividing the surface area by the volume.

For example, a cube has sides of 1 cm. The total surface area is the sum of the areas of each side. A cube has 6 equal sides, each side of this cube has an area of  $1 \text{ cm}^2$ . Therefore the surface area is  $1 \text{ cm}^2$ , multiplied by 6. So the surface area of the cube is  $6 \text{ cm}^2$ .

The volume of the cube is calculated by multiplying the length of each side:  
 $1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$

The surface area to volume ratio is 6 divided by 1 which equals 6. Note that ratios have no units.

The ratio is 6:1 but in this scenario it is normally expressed as a single number

Ratios are commonly used in genetics to represent, for example, phenotypic ratios for monohybrid and dihybrid crosses.

For example, in a fruit fly genetics experiment the offspring were counted; 78 fruit fly offspring had red eyes and 20 had brown eyes

Phenotype	Number of offspring
Red eyes	78
Brown eyes	20

To calculate the ratio of red-eyed flies to brown-eyed flies 78 is divided by 20, which equals 3.9.

So the ratio of red to brown is 3.9:1

If we now look at the ratio of brown-eyed flies to red-eyed flies, we would divide 20 by 78 which equals 0.256. This can be rounded up to 0.3. So the ratio is 0.3:1

When more than two quantities are all being compared the order in which the ratio is given follows the order in which the different quantities are named and the last one will always be given as 1, with the other numbers all relative to that.

For example, imagine there were four options for eye colour in flies: red, purple, scarlet and white, and the numbers of offspring are 127, 50, 32 and 8 respectively. The respective ratios are all relative to the last number. Therefore you divide each by the last number, which here is 8.

Phenotype	Red eyes	Purple eyes	Scarlet eyes	White eyes
Number of offspring	127	50	32	8
Calculating the ratio of offspring	$\frac{127}{8} = 15.9$	$\frac{50}{8} = 6.3$	$\frac{32}{8} = 4.0$	$\frac{8}{8} = 1$

The ratios of red to purple to scarlet to white is therefore 15.9 : 6.3 : 4.0 : 1

Again remember that with ratios there are no units.

## Scales for measuring

NB: This topic is covered thoroughly in sections M0.1 and M1.8.

Units show what a quantity is measured in, and generally a measured quantity without units is meaningless.

Learners would be expected to be able to convert between different units, as detailed in section M0.1, with respect to the scale they are measuring in.

For example, bacteria would generally be measured in micrometres ( $\mu\text{m}$ ), whereas an elephant would be measured in metres (m). As bacteria and elephants are nowhere near the same size, this example shows why understanding scales for measurements and the corresponding units are so important.

Being able to accurately label diagrams and pictures according to scale is essential, as detailed in section M1.8, and can be denoted by a sentence, ratio or scale bar.

## Maths skills – M0.3 Use ratios, fractions and percentages

### Quiz – Percentages: Practice calculations

Learners may be tested on their ability to:

- Calculate percentage yields
- Calculate surface area to volume ratio
- Use scales for measuring
- Represent phenotypic ratios (monohybrid and dihybrid crosses)

1. Ventricular systole lasts for 0.3 s. The cardiac cycle lasts for 0.8 s. What percentage of the cardiac cycle is ventricular systole?



2. In an onion root tip squash, 200 cells were observed and each cell was assigned to a stage of the cell cycle. Here are the results:

<b>Stage</b>	<b>Number of cells</b>
Interphase	150
Prophase	20
Metaphase	12
Anaphase	4
Telophase	8
Cytokinesis	6

What percentage of cells were at each stage of the cell cycle?

3. A soil sample weighed 2.4 g. After heating at 100°C in an oven to evaporate the water, it weighed 1.8 g. What percentage of the soil sample was water?

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4. Stearic acid has the formula  $C_{17}H_{35}COOH$ . What percentage of the atoms in stearic acid are:

(a) carbon?

(b) hydrogen?

(c) oxygen?


### Quiz – Percentage yield: Practice calculations

1. In the following examples you are given the actual yield and the theoretical yield. Calculate the percentage yield.

(a) Actual yield = 40 g    Theoretical yield = 60 g

(b) Actual yield = 60 g    Theoretical yield = 100 g

(c) Actual yield = 90 g    Theoretical yield = 130 g

(d) Actual yield = 23 g    Theoretical yield = 60 g


2. In the hydrolysis of a sample of triglycerides, the theoretical yield of fatty acids is 9.0 g. The actual yield was 7.2 g. What was the percentage yield for this synthesis?

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### Quiz – Ratio: Practice calculations

5. Calculate the surface area-to-volume ratios of the following cuboids:

(a) A cuboid with sides: 2 cm x 2 cm x 2 cm

(b) A cuboid with sides: 1 m x 2 m x 4 m

(c) A cuboid with sides: 1 mm x 1 mm x 8 mm


### Quiz – Phenotypic ratio: practice calculations

1. Plants were grown either in the light or the dark and the length of the stem was measured.

Growing conditions	Stem length (cm)
Light	10
Dark	25

a) What was the ratio of stem length, light to dark?

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b) What was the ratio of stem length, dark to light?

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2. The stem length experiment was repeated by growing plants under four different coloured lights:

Light used for growth	Stem length (cm)
Blue	25
Green	3
Yellow	10
Red	15

What was the ratio of stem length blue to green to yellow to red?

3. Let's say colour of naked mole rats is determined by a single gene and brown colour (B) is dominant to white colour (b). If two heterozygous (Bb) naked mole rats were mated, what is the expected ratio of brown naked mole rats to white naked mole rats?

## Maths skills – M0.4 Estimate results

### Tutorials

Learners may be tested on their ability to:

- Estimate results to sense check that the calculated values are appropriate.

It's very easy to write the wrong number or to press the wrong number on a calculator. Without having an estimation, your result can clearly be wrong but go unnoticed. You need to ask yourself, "Does that look right?". By comparing your calculated answer with your estimation it is easier to spot whether you have made a mistake.

For example, say we surveyed a small area of woodland and found 8 mice in  $10 \text{ m}^2$ . And we wanted a rough estimate of how many there might be in the woodland which was  $1 \text{ km}^2$ . We know that  $1 \text{ km}^2$  is equal to  $10^6 \text{ m}^2$  (see M0.1 if you are unsure about this bit) so we know that the answer will be many orders of magnitude higher than the number of 8 mice in  $10 \text{ m}^2$ . The sum is 8 mice  $\times 10^6 \text{ m}^2$  divided by  $10 \text{ m}^2$ . Which is 800,000 mice in  $1 \text{ km}^2$ . If you put the numbers into the calculator in the wrong order, or divide when you should be multiplying, you know that a number that has not got five or six zeros after it, is wrong.

In this example you could work out the number without a calculator but what if you found 8 mice in  $14 \text{ m}^2$  and the woodland had an area of  $2.3 \text{ km}^2$ .

Knowing that there are  $10^6 \text{ m}^2$  in a  $\text{km}^2$  tells you that the answer will be around  $10^6$  mice.

The calculation is 8 mice  $\times 2.3 \times 10^6 \text{ m}^2$ , divided by  $14 \text{ m}^2$ . The answer is 1,314,286, rounded to the nearest whole mouse. This is better communicated in standard form (see M0.2 if you are not sure about this) which gives  $1.3 \times 10^6$  mice.

The key message is to think about your answer, and have a rough idea of its order of magnitude before you get out your calculator.

Sometimes the best way to make estimations in calculations is to round numbers (particularly decimals) up or down to a number you're happier working with. For example, the calculation:  $4.9/11.2$  could be estimated as  $5/10 = 0.5$

If you get the answer 54.88 then the estimate shows that you have done something wrong – in this case you would have pressed the multiply key instead of the divide key on your calculator.

Estimating is a useful skill for any biologist and can be used in all sorts of contexts – from the working out the number of bacteria in a dilution, to measuring lengths when using a microscope, to ecological sampling.

Think about your answer, and ask yourself, "Does this answer make sense?". I cannot stress enough how important this is, in all calculations you do.

## Maths skills – M0.4 Estimate results

### Quiz

1. Estimate these and check your approximations using a calculator

a)  $15.6 \times 24.4$

b)  $8.8 \times 32.3$

c)  $15.6/24.4$

d)  $8.8/32.3$


2. Here's an example from ecological sampling. A 0.5 m x 0.5 m quadrat was used to count plants. Ten samples were taken and the plants in each quadrat counted.

The values were: 7, 3, 6, 12, 4, 8, 8, 9, 10, 2

Make an estimate of the density of plants per m<sup>2</sup>.

Now calculate the density of plants per m<sup>2</sup>.

3. Estimate whether the following calculations will give an answer that is greater than, or less than, one.

a)  $\frac{3.872}{5 \times 10.2}$

b)  $\frac{3}{6 \times 0.2}$

c)  $\frac{452 \times 7}{84 \times 113}$

d)  $\frac{0.5 \times 0.2}{0.01}$

## Maths skills – M0.5 Using calculators to find and use power, exponential and logarithmic functions

### Tutorials

Learners may be tested on their ability to:

- estimate the number of bacteria grown over a certain length of time.

Note: this is an A Level mathematical skill, not assessed at AS Level

Different models of calculators have different ways of entering powers and using them. Some examples of different calculator symbols include: 'x<sup>n</sup>', '10<sup>x</sup>', '^' and 'exp'.

But let's look at logs first. Logarithms (or logs) can have any positive value as their base, but the two bases used in biology are 10, referred to as common logs, and the base 2.71828...etc, the value for e, referred to as natural logs. On a calculator, log to the base 10 is the button with log written on it, which can be written as "*log(x)*". Log to the base e has the calculator button ln and is written as "*ln(x)*".

Common log	Natural log
Base-10	Base-e
<i>log(x)</i>	<i>ln(x)</i>
log	ln

You need to know how to use the natural log and common log buttons.

Have a go at working out both the natural and common logs of  $3 \times 10^5$ .

You should end up with 5.477 for common logs and 12.611 for natural logs.

You also need to know how to use the exponential function on your calculator. You may be expected to estimate the number of bacteria grown over a certain length of time. Bacterial cells multiply exponentially. If a culture of bacteria increases by a factor of 2 (which means the population doubles) in one generation, then the formula to find the number of cells, N, after *n* generations is:

$$N = N_0 \times 2^n \quad N_0 = \text{initial number} \quad n = \text{number of divisions, or generations}$$

$N_0$ , the initial number at time zero, multiplied by 2 to the power little n where little n is the number of divisions, or generations..

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For example, one bacterial cell will divide about every 20 min under standard conditions.

The cells produced will form a sequence of numbers:

Division 0	Division 1	Division 2	Division 3	Division 4	Division 5	Division 6	Division 7
1	2	4	8	16	32	64	128

For example, after the 6<sup>th</sup> division there will be 64 cells. The formula to calculate the number in the  $n$ th generation is given by the formula  $N = N_0 \times 2^n$  where  $n$  is the number of generations.

So, how many cells will there be after 4 hours?

You have to work out how many divisions there have been in 4 hours. The bacteria divide once every 20 minutes, which is three times an hour. Therefore, in 4 hours the bacteria will divide 12 times. Little  $n$  will be equal to 12. You then put this into the original equation.

You started with one bacterial cell, so  $N_0=1$

So the number of cells, capital  $N$ , is equal to  $1 \times 2$  to the power 12. Which means there are 4096 bacterial cells after 4 hours

60 minutes / 20 minutes = 3 divisions an hour

4 x 3 = 12 divisions

$$N = N_0 \times 2^n = 1 \times 2^{12} = 4096 \text{ bacterial cells}$$

To do this calculation successfully you need to know how to enter exponential powers into your calculator.

It is important for you to get to know how to put numbers into your calculator to get the right answers out of it.



## Maths skills – M0.5 Use calculators to find and use power, exponential and logarithm functions

### Quiz

1. Use your calculator to determine the value of:

a)  $12^3$

b)  $10^5$

c)  $\text{Log}(4)$

d)  $\text{Ln}(7)$

2. A bacterial cell divides (doubles) every 45 minutes under standard conditions. There is initially one bacterium in the culture. How many cells will there be after 6 hours?

3. A bacterial cell divides (doubles) every 35 minutes under standard conditions. There is initially one bacterium in the culture. How many cells will there be after 8 hours?

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