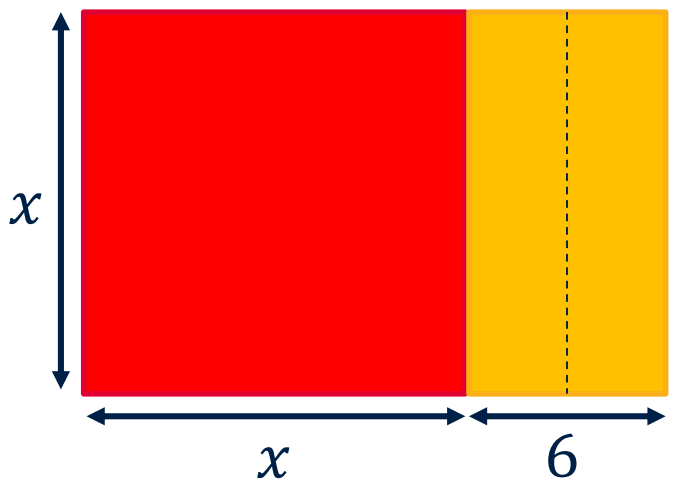
Advanced Mathematics Support Programme®

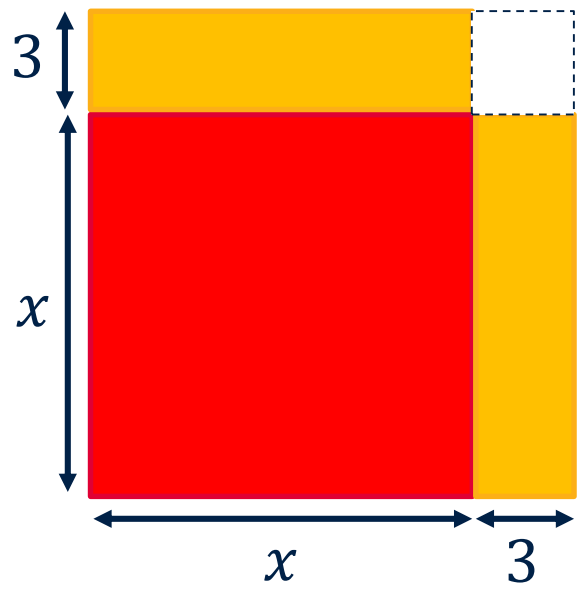
Did you know?

These are different forms of the same algebraic expression

$$x^2 + 6x = x(x+6) = (x+3)^2 - 9$$

expanded form | factorised form | completed square form



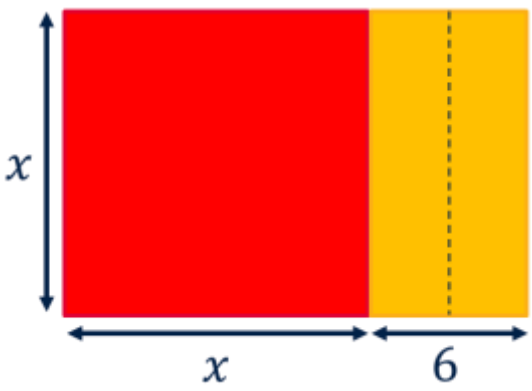


Do the diagrams help you see why this is called Completing the square?

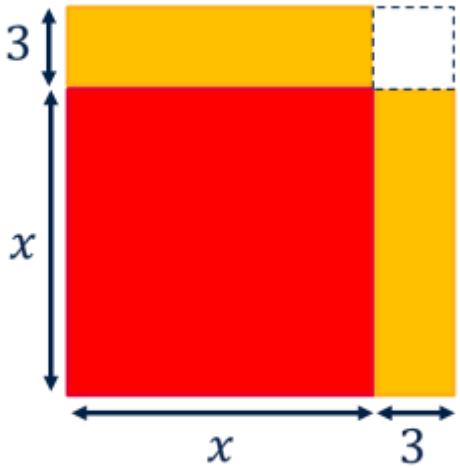
Something similar

Think you've seen these diagrams before?

- They are very similar to the diagrams for the Difference of Two Squares – *as seen previously in Expanding Double Brackets.*



$$x^2 + 6x$$



Can you see this is the difference of two squares?

$$(x+3)^2 - 3^2$$

$$\big((x+3) - 3\big)\big((x+3) + 3\big)$$

Collect like terms within the brackets

$$= x(x+6)$$

$$= x^2 + 6x$$

Completing the square 1

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 4x$
2. $x^2 + 4x + 5$
3. $y^2 - 8y$
4. $y^2 - 8y + 7$
5. $x^2 - 12x + 41$
6. $k^2 + 10k - 2$
7. $y^2 + 3y + 1$
8. $p^2 - 2p + 1$

Completing the square 1

Solutions on the next slide….

Completing the square 1

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 4x$ → $= (x + 2)^2 - 4$

2. $x^2 + 4x + 5$ → $= (x + 2)^2 + 1$

3. $y^2 - 8y$ → $= (y - 4)^2 - 16$

4. $y^2 - 8y + 7$ → $= (y - 4)^2 - 9$

Unsure about any of these? Search Completing the square. Next try Skills check 2….

Completing the square 1

Write these expressions in the form $(x+a)^2+b$

5. $x^2-12x+41$ → $=(x-6)^2+5$

6. $k^2+10k-2$ → $=(k+5)^2-27$

7. y^2+3y+1 → $=\left(y+\frac{3}{2}\right)^2-\frac{5}{4}$

8. p^2-2p+1 → $=(p-1)^2$

Unsure about any of these? Search Completing the square. Next try Skills check 2….

Completing the square 2

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 10x$
2. $x^2 + 10x + 30$
3. $y^2 - 2y$
4. $y^2 - 2y + 3$
5. $x^2 - 8x + 25$
6. $k^2 + 14k - 1$
7. $y^2 + 5y + 6$
8. $t^2 + 6t + 9$

You can do this for fun - or move on if you correctly completed Skills check 1.

Completing the square 2

Solutions on the next slide….

Completing the square 2

Write these expressions in the form $(x + a)^2 + b$

1. $x^2 + 10x$ → $= (x + 5)^2 - 25$

2. $x^2 + 10x + 30$ → $= (x + 5)^2 + 5$

3. $y^2 - 2y$ → $= (y - 1)^2 - 1$

4. $y^2 - 2y + 3$ → $= (y - 1)^2 + 2$

Completing the square 2

Write these expressions in the form $(x+a)^2+b$

5. $x^2-8x+25$ → $=(x-4)^2+9$

6. $k^2+14k-1$ → $=(k+7)^2-50$

7. y^2+5y+6 → $=\left(y+\frac{5}{2}\right)^2-\frac{1}{4}$

8. t^2+6t+9 → $=(t+3)^2$

Different forms

It is important to be able to convert expressions between the different forms:

expanded form | factorised form | completed square form

In this problem there are 4 sets of three equivalent expressions, however, some expressions are missing. Match the sets and find the 3 missing expressions.

$a^2 - 2a - 8$		$a^2 - 8a + 15$
	$a^2 + 2a - 15$	$(a + 2)(a + 4)$
$(a + 1)^2 - 16$	$(a - 3)(a - 5)$	
$(a + 5)(a - 3)$	$(a - 1)^2 - 9$	$(a + 3)^2 - 1$

Different forms

Solutions on the next slide….

Different forms Solutions

$a^2 - 2a - 8$	$(a-4)^2 - 1$	$a^2 - 8a + 15$
$a^2 + 6a + 8$	$a^2 + 2a - 15$	$(a+2)(a+4)$
$(a+1)^2 - 16$	$(a-3)(a-5)$	$(a-4)(a+2)$
$(a+5)(a-3)$	$(a-1)^2 - 9$	$(a+3)^2 - 1$

Extra Puzzles

What is the value of

$$\frac{\frac{(5^2-3^2)}{5+3}+\frac{(4^2-2^2)}{4+2}+\frac{(3^2-1^2)}{3+1}}{2}\ ?$$

Extra Puzzles

Given that

$$55^2 - 45^2 = (55 + 45)(55 - 45) = 1000$$

and

$$60^2 - 40^2 = (60 + 40)(60 - 40) = 2000$$

Find numbers a and b such that $a^2 - b^2 = 3000$

Find numbers c and d such that $c^2 - d^2 = 4000$

Find numbers e and f such that $e^2 - f^2 = 100\,000$

Extra Puzzles

Follow the link to the solutions

Still want more?

Learn about the history of solving quadratics and completing the square by learning about an Arab mathematician who is considered to be the founder of algebra.

Discover about removing cubes rather than squares. Does this activity help you consider the challenges involved in ‘completing the cube’?.

Watch this clip on parabolic flight. Think about the information you have learnt from completing the square and factorising, and how that links to the parabolic flight.

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