



**Advanced Mathematics  
Support Programme®**

## Maths can be murderous!

You will have heard of **Pythagoras** and his theorem but have you heard of **Hippasus** who was one of his followers?

Pythagoreans preached that all numbers could be expressed as the ratio of integers – i.e. fractions.

Hippasus is sometimes credited with the discovery of the existence of irrational numbers – proving it for  $\sqrt{2}$ .

Following which, he was drowned at sea!





1. Simplify  $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

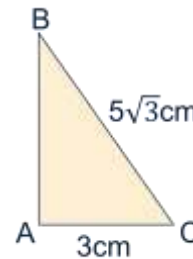
2. Simplify  $\sqrt{2} \times \sqrt{6}$

3. Simplify fully  $(4\sqrt{3})^2$

4. Write  $\sqrt{45} + \sqrt{20}$  in the form  $k\sqrt{5}$

5. Calculate  $\frac{\sqrt{54}}{\sqrt{6}}$

6. Rationalise the denominator of  $\frac{4}{\sqrt{3}}$



7. Find the length AB

8. A rectangle has an area of  $8\sqrt{15}$  cm<sup>2</sup> and a length of  $2\sqrt{3}$  cm.

Find the width of the rectangle



# Surds 1



Solutions on the next slide....



1. Simplify  $\sqrt{a} + 2\sqrt{a} + 5\sqrt{a}$

→ 1.  $= 8\sqrt{a}$

Here we are just using the same skills here as when collecting like terms with algebraic expressions e.g.  $x + 2x + 5x = 8x$

2. Simplify  $\sqrt{2} \times \sqrt{6}$

→ 2.  $= \sqrt{2 \times 6} = \sqrt{12}$   
 $= \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

3. Simplify fully  $(4\sqrt{3})^2$

→ 3.  $= 4\sqrt{3} \times 4\sqrt{3}$   
 $= 4 \times 4 \times \sqrt{3} \times \sqrt{3}$   
 $= 16 \times 3$   
 $= 48$

4. Write  $\sqrt{45} + \sqrt{20}$  in the form  $k\sqrt{5}$

→ 4.  $\sqrt{45} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$   
 $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$   
 $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$

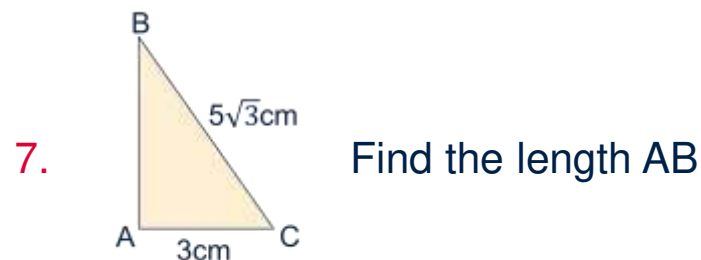


5. Calculate  $\frac{\sqrt{54}}{\sqrt{6}}$

→ 5.  $\frac{\sqrt{54}}{\sqrt{6}} = \frac{\sqrt{9 \times 6}}{\sqrt{6}} = \frac{\sqrt{9} \times \sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{\sqrt{6}} = 3$

6. Rationalise the denominator of  $\frac{4}{\sqrt{3}}$

→ 6.  $\frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{4\sqrt{3}}{3}$



→ 7. Using Pythagoras' theorem  
 $AB^2 = (5\sqrt{3})^2 - 3^2$   
 $AB^2 = (25 \times 3) - 9$   
 $AB^2 = 66$  so  $AB = \sqrt{66}$  cm

8. A rectangle has an area of  $8\sqrt{15}$  cm<sup>2</sup> and a length of  $2\sqrt{3}$  cm.

→ 8.  $8\sqrt{15} = \text{width} \times 2\sqrt{3}$   
 $8\sqrt{15} \div 2\sqrt{3} = \text{width}$   
 $\frac{8\sqrt{15}}{2\sqrt{3}} = \frac{8 \times \sqrt{5} \times \sqrt{3}}{2\sqrt{3}} = 4\sqrt{5}$  cm

Find the width of the rectangle



1. Simplify  $\sqrt{d} + 6\sqrt{d} - 3\sqrt{d}$

2. Simplify  $2\sqrt{b} \times 4\sqrt{3}$

3. Simplify fully  $(4\sqrt{5})^2$

4. Write  $\sqrt{75} + \sqrt{48} - 2\sqrt{12}$   
in the form  $k\sqrt{3}$

5. Simplify  $\frac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$

6. Rationalise the denominator of  $\frac{2\sqrt{2}}{\sqrt{5}}$

7. Evaluate  $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$

Give your answer in simplest form.  
Rationalise the denominator.

8. A triangle has a base of  $3\sqrt{2}$  and a perpendicular height of  $5\sqrt{8}$ .  
Calculate the area of the triangle.



## Surds 2



Solutions on the next slide....





1. Simplify  $\sqrt{d} + 6\sqrt{d} - 3\sqrt{d}$

$$\rightarrow = 4\sqrt{d}$$

2. Simplify  $2\sqrt{b} \times 4\sqrt{3}$

$$\begin{aligned} \rightarrow &= 2 \times 4 \times \sqrt{b} \times \sqrt{3} \\ &= 8 \times \sqrt{b \times 3} \\ &= 8\sqrt{3b} \end{aligned}$$

3. Simplify fully  $(4\sqrt{5})^2$

$$\begin{aligned} \rightarrow &= 4\sqrt{5} \times 4\sqrt{5} \\ &= 4 \times 4 \times \sqrt{5} \times \sqrt{5} \\ &= 16 \times 5 \\ &= 80 \end{aligned}$$

4. Write  $\sqrt{75} + \sqrt{48} - 2\sqrt{12}$   
in the form  $k\sqrt{3}$

$$\begin{aligned} \rightarrow \sqrt{75} &= \sqrt{25} \times \sqrt{3} = 5\sqrt{3} \\ \sqrt{48} &= \sqrt{16} \times \sqrt{3} = 4\sqrt{3} \\ 2\sqrt{12} &= 2 \times \sqrt{4} \times \sqrt{3} = 2 \times 2\sqrt{3} = 4\sqrt{3} \\ \\ &5\sqrt{3} + 4\sqrt{3} - 4\sqrt{3} = 5\sqrt{3} \end{aligned}$$



5. Simplify  $\frac{\sqrt{125} - 2\sqrt{20}}{\sqrt{5}}$   $\rightarrow$  
$$= \frac{\sqrt{25}\sqrt{5} - 2\sqrt{4}\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5} - 2 \times 2\sqrt{5}}{\sqrt{5}}$$

$$= \frac{5\sqrt{5} - 4\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5}} = 1$$

6. Rationalise the denominator of  $\frac{2\sqrt{2}}{\sqrt{5}}$   $\rightarrow$  
$$\frac{2\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{2}\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{2\sqrt{10}}{5}$$

7. Evaluate  $\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$   $\rightarrow$  
$$= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}} = \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{6}}$$

Need a common denominator to add fractions

$$= \frac{1 \times (\sqrt{3})}{\sqrt{2} \times (\sqrt{3})} + \frac{\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{6}} + \frac{\sqrt{3}}{\sqrt{6}} = \frac{2\sqrt{3}}{\sqrt{6}}$$

$$= \frac{2\sqrt{3}}{\sqrt{2}\sqrt{3}} = \frac{2}{\sqrt{2}} = \frac{2 \times (\sqrt{2})}{\sqrt{2} \times (\sqrt{2})} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Give you answer in simplest form.  
Rationalise the denominator.

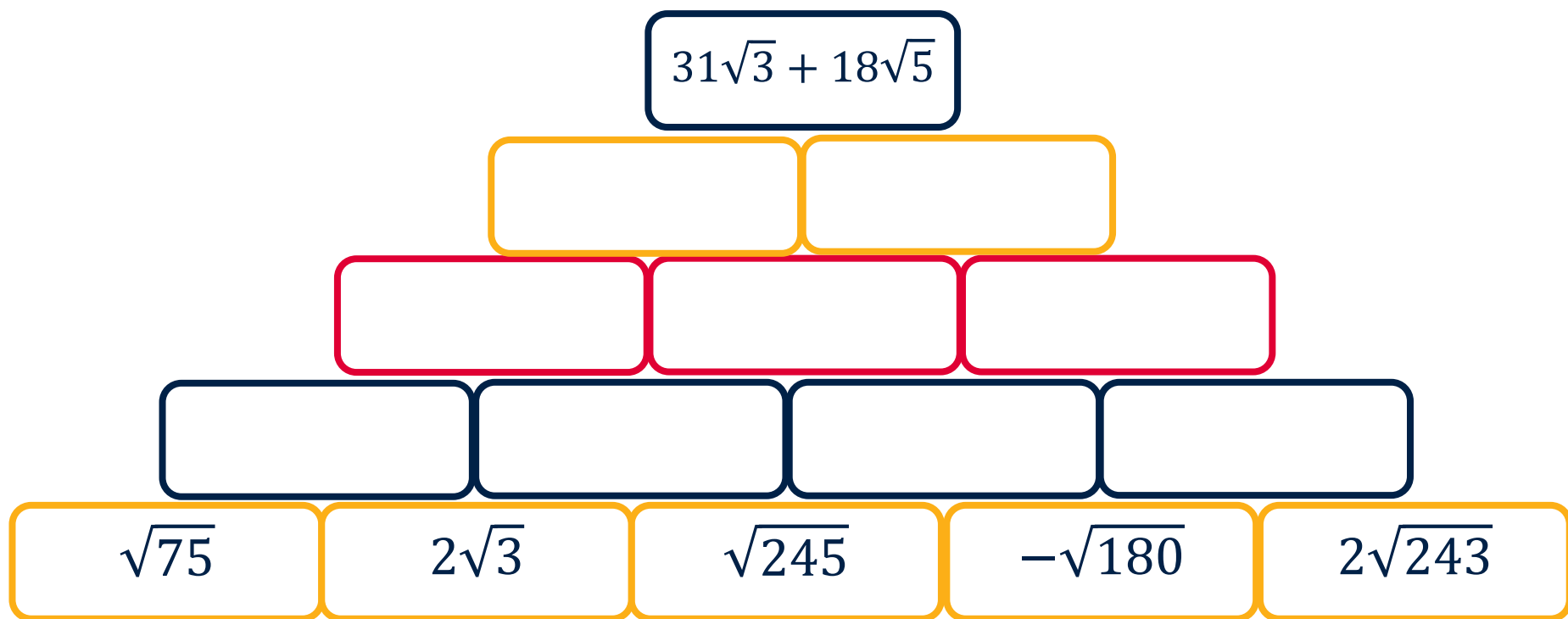
8. A triangle has a base of  $3\sqrt{2}$  and a perpendicular height of  $5\sqrt{8}$ . Calculate the area of the triangle.  $\rightarrow$  
$$\text{Area} = \frac{1}{2} \times 3\sqrt{2} \times 5\sqrt{8} = \frac{1}{2} \times 3 \times 5 \times \sqrt{2}\sqrt{8}$$

$$= \frac{1}{2} \times 15 \times \sqrt{16} = \frac{1}{2} \times 15 \times 4$$

$$= 30 \text{ cm}^2$$



Complete the empty boxes in the pyramid.  
Each box is the sum of the two boxes directly below it.



\*Hint: You may need to simplify some of the surds in the bottom row to get started.

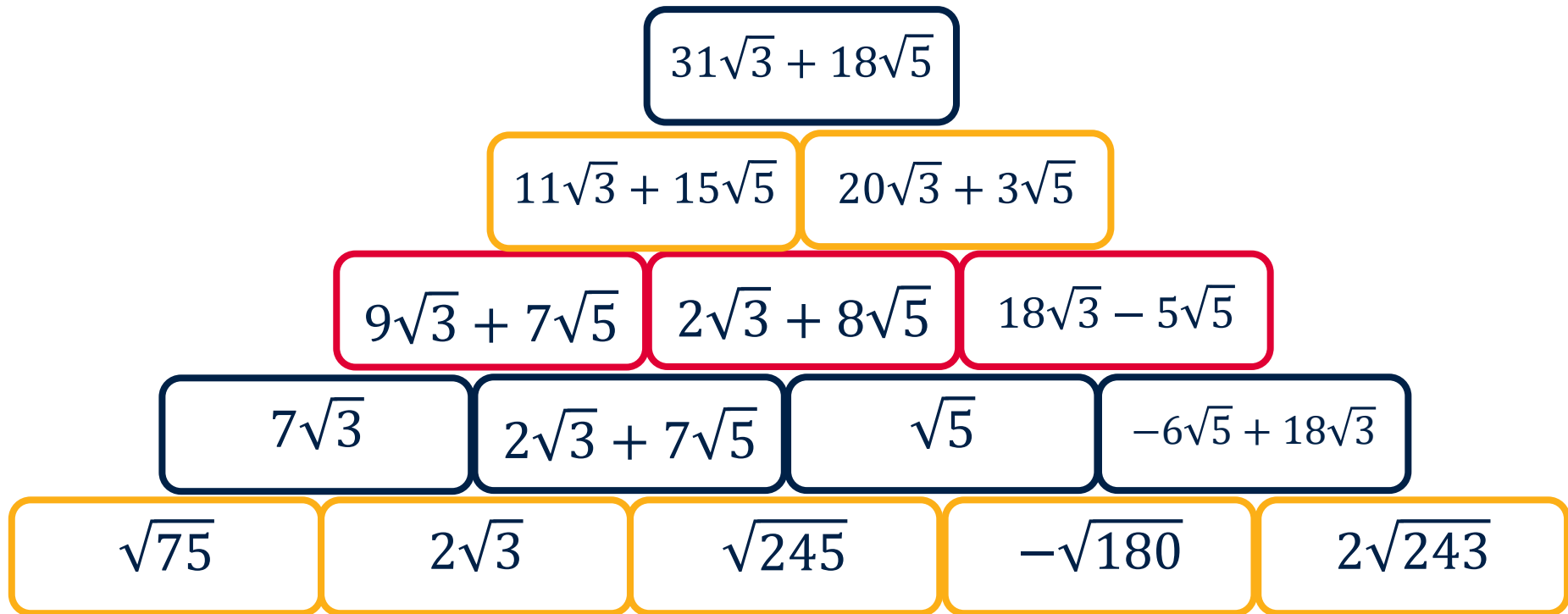
# Another brick in the wall



Solutions on the next slide....



Complete the empty boxes in the pyramid.  
Each box is the sum of the two boxes directly below it.





Decide if each of the following expressions is **True** or **False**

1.  $\sqrt{9} + \sqrt{4} = \sqrt{13}$

5.  $\frac{\sqrt{12} \times \sqrt{3}}{\sqrt{9}} = 2$

2.  $\sqrt{a} \times \sqrt{b} = \sqrt{c}$

6.  $\sqrt{2}^3 = 2\sqrt{2}$

3.  $\sqrt{(8)^2} = 8$

7.  $\sqrt{ab}^2 = ab$

4.  $10\sqrt{2} = 5\sqrt{8}$

8.  $2\sqrt{100} = \sqrt{200}$

Are there any statements that are sometimes true but not always true?

Can you explain why?

True or False?



Solutions on the next slide....



Decide if each of the following expressions is **True** or **False**

1.  $\sqrt{9} + \sqrt{4} = \sqrt{13}$     **False**     $\rightarrow$      $\sqrt{9} = 3$  and  $\sqrt{4} = 2$   
 $3 + 2 = 5$  and  $\sqrt{13} \neq 5$

2.  $\sqrt{a} \times \sqrt{b} = \sqrt{c}$     **True**     $\rightarrow$     when  **$a \times b = c$**  e.g.  $a=5, b=6, c=30$   
**False**     $\rightarrow$     otherwise

3.  $\sqrt{(8)^2} = 8$     **True**     $\rightarrow$      $\sqrt{(8)^2} = \sqrt{64} = 8$

4.  $10\sqrt{2} = 5\sqrt{8}$     **True**     $\rightarrow$      $10\sqrt{2} = \sqrt{100} \times \sqrt{2} = \sqrt{200}$   
 $5\sqrt{8} = \sqrt{25} \times \sqrt{8} = \sqrt{200}$





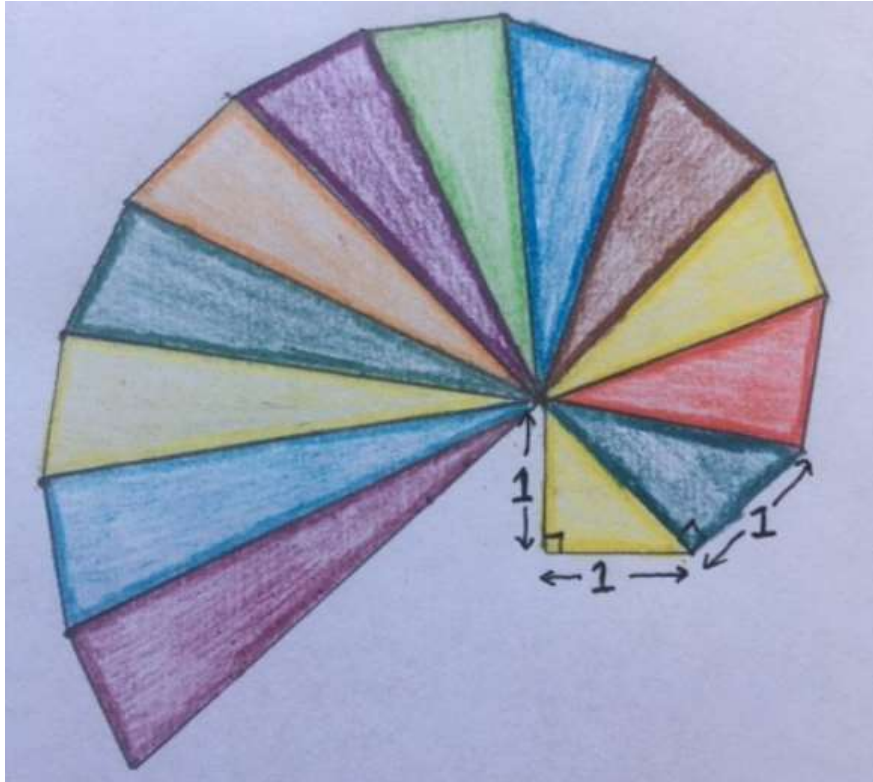
Decide if each of the following expressions is **True** or **False**

$$5. \quad \frac{\sqrt{12} \times \sqrt{3}}{\sqrt{9}} = 2 \quad \text{True} \quad \rightarrow \quad \frac{\sqrt{12} \times \sqrt{3}}{\sqrt{9}} = \frac{\sqrt{36}}{\sqrt{9}} = \sqrt{\frac{36}{9}} = \sqrt{4}$$

$$6. \quad \sqrt{2}^3 = 2\sqrt{2} \quad \text{True} \quad \rightarrow \quad \begin{aligned} \sqrt{2}^3 &= \sqrt{2} \times \sqrt{2} \times \sqrt{2} \\ &= \sqrt{4} \times \sqrt{2} = 2\sqrt{2} \end{aligned}$$

$$7. \quad \sqrt{ab}^2 = ab \quad \text{True} \quad \rightarrow \quad \sqrt{ab}^2 = \sqrt{ab} \times \sqrt{ab} = ab$$

$$8. \quad 2\sqrt{100} = \sqrt{200} \quad \text{False} \quad \rightarrow \quad 2\sqrt{100} = \sqrt{4} \times \sqrt{100} = \sqrt{400}$$



The diagram shows a spiral made up of right-angled triangles.

The shortest side of each triangle measures 1 unit.

Can you see how it is constructed?

Find the length of the hypotenuse for the first few triangles.

What do you notice?

Which triangles would have a side of length 3?

What other questions could you ask about the diagram?

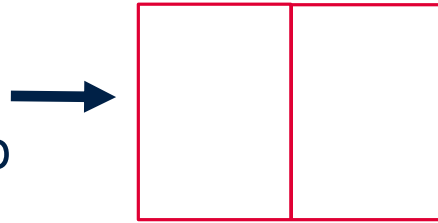
# Wheel of Theodorus



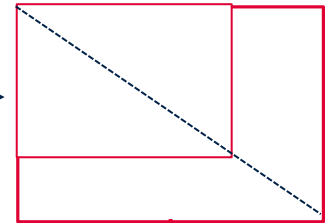
Follow the [link](#) to the solutions



Actually take two sheets and place them side by side to make a piece of A3.

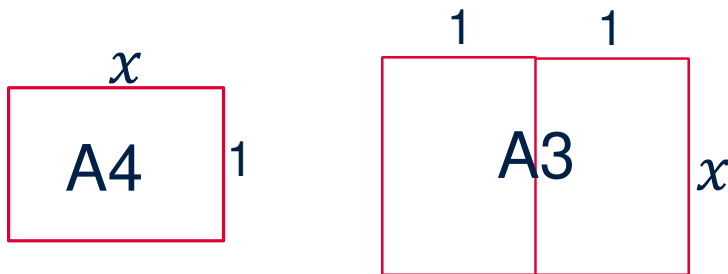


Then place another sheet of A4 on top of it like this:



What do you notice about the ratio of the sides of an A3 sheet compared with the A4 sheet?

Thinking about the ratio of the long side to the short side we get:



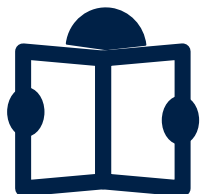
$$\frac{x}{1} = \frac{2}{x} \rightarrow x^2 = 2 \rightarrow x = \sqrt{2}$$

Therefore, for A4, A3, A2, etc... the length of the long side divided by the length of the short side is always  $\sqrt{2}$

Take a sheet of A4 paper



Follow the [link](#) to the solutions



Read about how **Irrational numbers** can “Inspir-al” you!  
It’s where mathematics and art meet!



Discover the proof, that  $\sqrt{2}$  is irrational – without getting murdered like Hippasus.



Watch this video to find out more about the special properties of A4 paper and discover what makes  $\sqrt{2}$  one of the most popular surds of all time.

# Contact the AMSP



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