



**Advanced Mathematics
Support Programme®**

Trigonometric functions can be used to model many things that repeat over a time period, for example



Tides



Springs



Harmonic Strings



Daylight



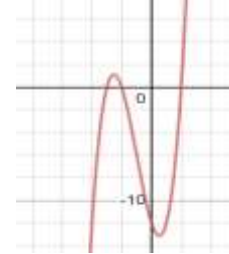
1. What is the mathematical name for the graph of $y = \frac{1}{x}$?

2. What are the maximum and minimum values for the graph $y = \cos\theta$?

3. Sketch the graph of $y = 2^x$.
Label the y and x intercepts

4. Using a sketch of the graphs
 $y = \frac{1}{x}$ and $y = x$
show how many solutions there will be to
the equation $\frac{1}{x} = x$

5. What is the name for this type of graph?



6. What is the y intercept of the graph
 $y = (x + 2)(x - 3)(x + 5)$?

7. What are the x intercepts of the graph
 $y = (x + 2)(x - 3)(x + 5)$?

8. Sketch the graph of
 $y = (x - 3)(x + 2)(x + 5)$



Sketching Other Graphs 1



Solutions on the next slide....



1. What is the mathematical name for the graph of $y = \frac{1}{x}$?



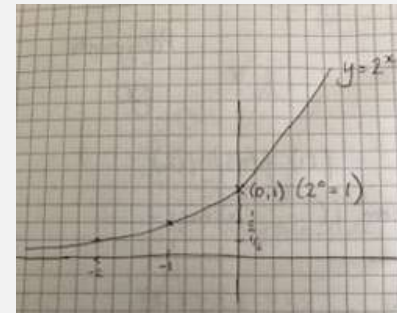
A reciprocal graph

2. What are the maximum and minimum values for the graph $y = \cos\theta$?



Max value = 1
Min value = -1

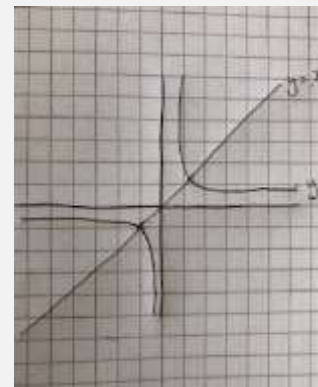
3. Sketch the graph of $y = 2^x$.
Label the y and x -intercepts



As x gets very large
 y gets very large

As x gets very small,
 y tends to zero but
stays positive

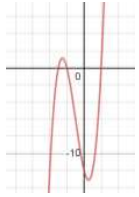
4. Using a sketch of the graphs
 $y = \frac{1}{x}$ and $y = x$
show how many solutions there will be to
the equation $\frac{1}{x} = x$



There will be
two solutions



5. What is the name for this type of graph? →



A cubic

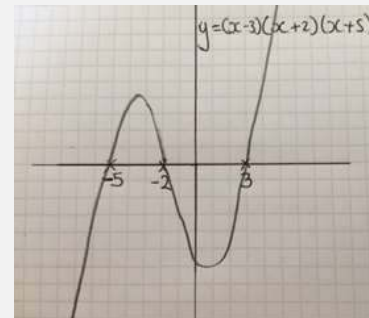
6. What is the y intercept of the graph
 $y = (x + 2)(x - 3)(x + 5)$? →

y intercept when $x = 0$
 $y = (0 + 2)(0 - 3)(0 + 5)$
 $y = 2 \times (-3) \times 5$
 $y = -30$
 y intercept at $(0, -30)$

7. What are the x intercepts of the graph
 $y = (x + 2)(x - 3)(x + 5)$? →

x intercepts when $y = 0$
 $0 = (x + 2)(x - 3)(x + 5)$
 $x = -2$ and $x = 3$ and $x = -5$
 x intercepts at $(-2, 0)$ $(3, 0)$ $(-5, 0)$

8. Sketch the graph of
 $y = (x - 3)(x + 2)(x + 5)$ →





1. What is the mathematical name for graphs of the form of $x^2 + y^2 = 9$?
2. Sketch the graph of $y = \sin\theta$ between 0° and 360° , labelling x and y intercepts
3. On your sketch for Q2 draw in the line $y = 0.5$
How many solutions are there to $\sin\theta = 0.5$?
Can you say what they are?
4. Sketch the graph $y = x^3$, labelling any intersections
5. Sketch the graph of the equation in Q1, label any intersections with the x and y axis
6. What is the y intercept of the graph $y = (x + 1)(x + 1)(x - 1)$?
7. What are the x intercepts of the graph $y = (x + 1)(x + 1)(x - 1)$?
8. Sketch the graphs of $x^2 + y^2 = 4$
 $y = x + 1$
Use the sketch to determine how many solutions there are when those equations are solved simultaneously



Sketching Graphs 2



Solutions on the next slide....

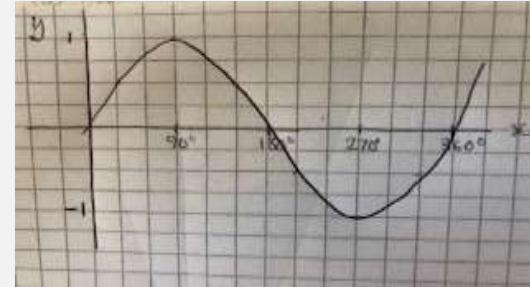


1. What is the mathematical name for graphs of the form of $x^2 + y^2 = 9$?

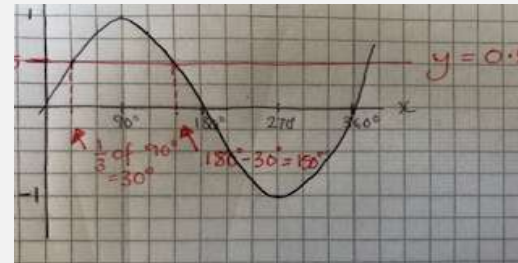


Circles are of the form $x^2 + y^2 = r^2$

2. Sketch the graph of $y = \sin\theta$ between 0° and 360° , labelling x and y intercepts

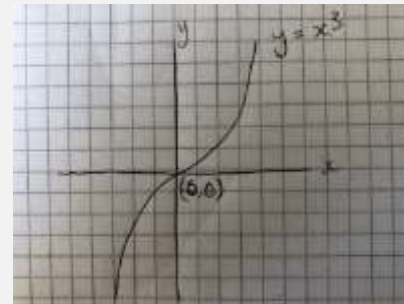


3. On your sketch for Q2 draw in the line $y = 0.5$
How many solutions are there to $\sin\theta = 0.5$?
Can you say what they are?



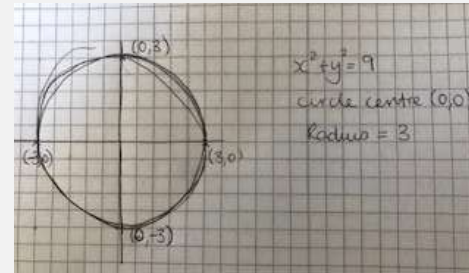
Two solutions
 30° and 150°
The points of intersection

4. Sketch the graph $y = x^3$, labelling any intersections





5. Sketch the graph of the equation in Q1. Label any intersections with the x and y axis



6. What is the y -intercept of the graph $y = (x + 1)(x + 1)(x - 1)$?



y intercept when $x = 0$

$$y = (0 + 1)(0 + 1)(0 - 1)$$

$$y = 1 \times 1 \times -1$$

$$y = -1$$

y intercept is $(0, -1)$

7. What are the x intercepts of the graph $y = (x + 1)(x + 1)(x - 1)$?



x intercept when $y = 0$

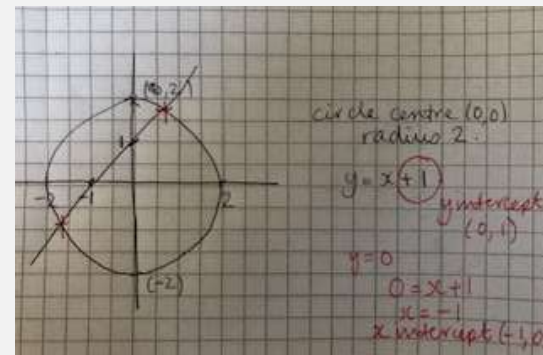
$$0 = (x + 1)(x + 1)(x - 1)$$

$$x = -1 \quad x = 1$$

x intercept is $(-1, 0)$ repeated and $(1, 0)$

8. Sketch the graphs of $x^2 + y^2 = 4$
 $y = x + 1$

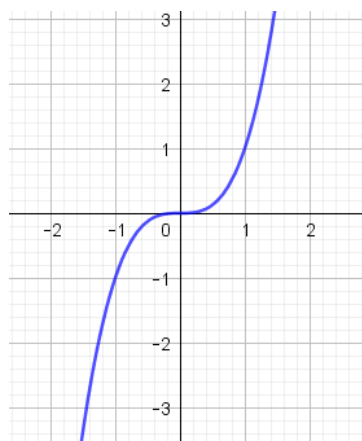
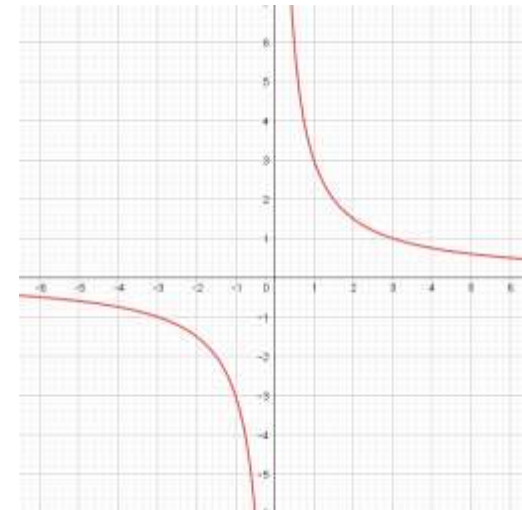
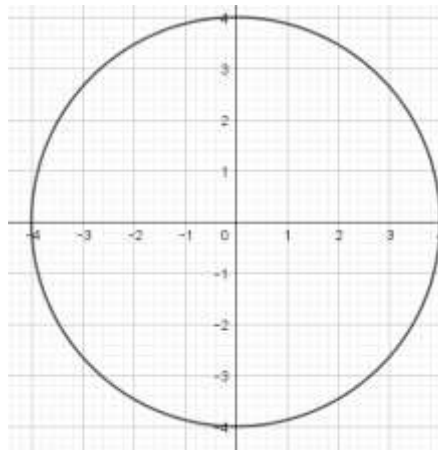
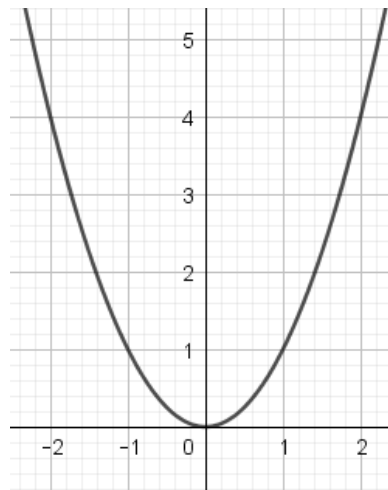
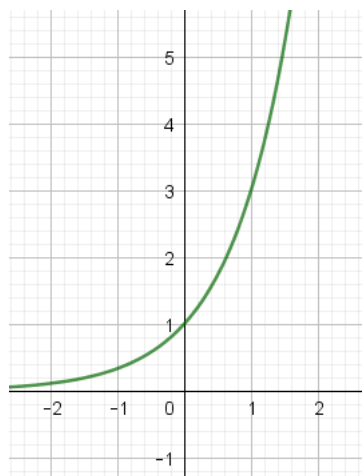
Use the sketch to determine how many solutions there are when those equations are solved simultaneously



Two solutions



Match the graphs to the equations **There are more equations than you need!**



- $y = 3^x$
- $y = \frac{3}{x}$
- $y = \sin\theta$
- $y = 2x^2$
- $y = \tan\theta$

- $y^2 + x^2 = 16$
- $y = x^2$
- $y = x^3$
- $y = \frac{1}{x}$
- $y = x^2 + 3$

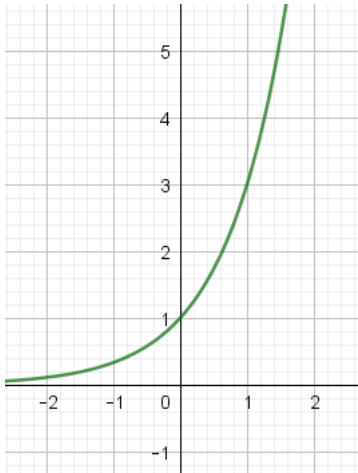
Which is which?



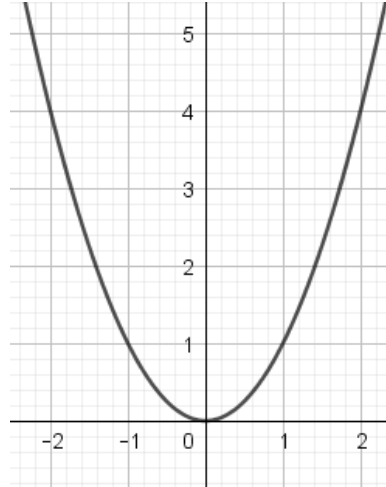
Solutions on the next slide....



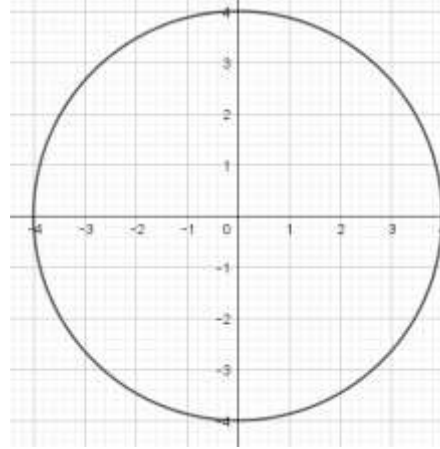
Match the graphs to the equations



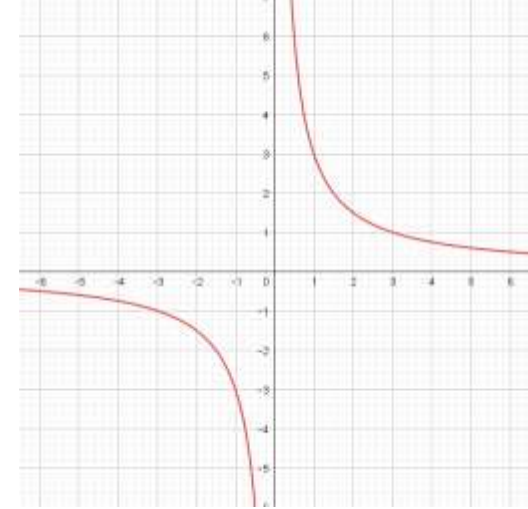
$$y = 3^x$$



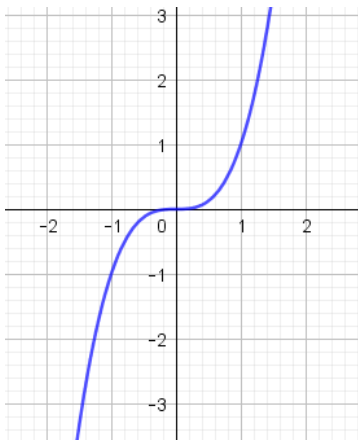
$$y = x^2$$



$$y^2 + x^2 = 16$$



$$y = \frac{3}{x}$$



$$y = x^3$$



$$y = \sin(x)$$



Find the shortest distance between the following curves:

$$x^2 + y^2 = 9$$

$$y = x^2 + 7$$

A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400m/min. It travels at this speed for 3 minutes before slowing to a stop via constant deceleration over a period of 30 seconds.

- What is the car's average speed for the first 20 seconds of travel?
- What is the car's deceleration?

A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.

Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.

What is the ratio of the lengths of the radius of C_1 and the radius of C_2 ?

Hint: Assume C_2 has a radius of 1 unit

Sketching more than graphs



Solutions on the next slides....



Find the shortest distance between the following curves:

$$x^2 + y^2 = 9$$

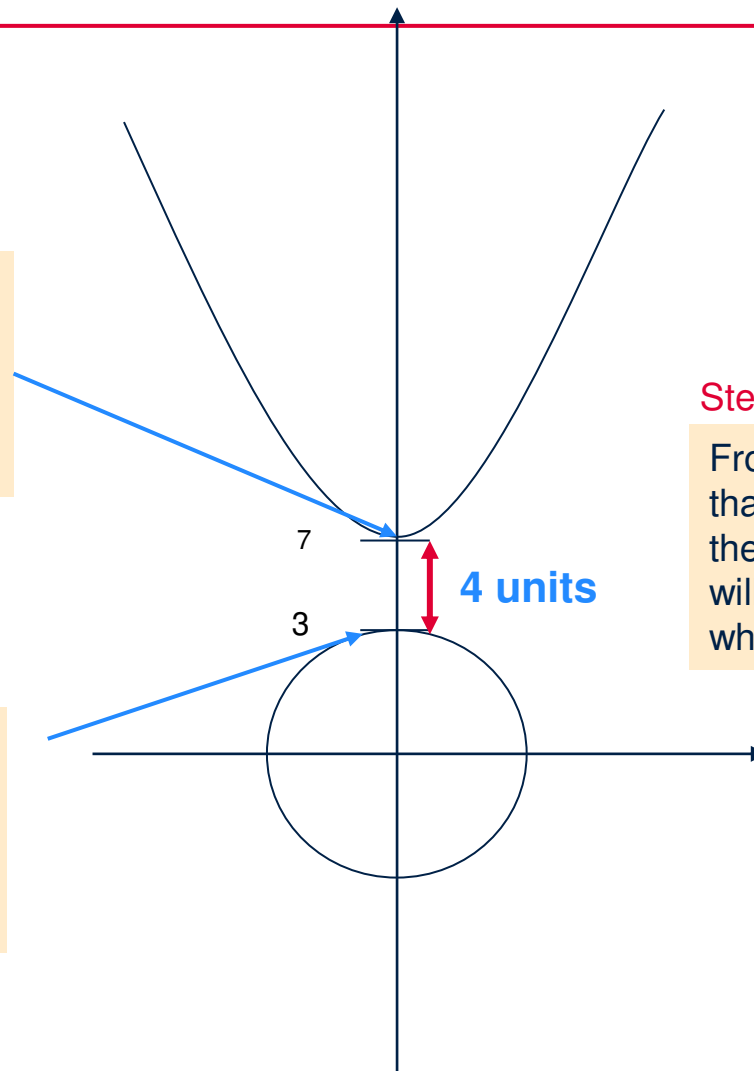
$$y = x^2 + 7$$

Step 1

The equation $y = x^2 + 7$ is a quadratic and is the equation $y = x^2$ shifted up the y axis by 7 units so lowest point will be at $(0,7)$

Step 2

The equation $x^2 + y^2 = 9$ is a circle with radius 3 and centre $(0,0)$ so sketch circle that goes through the points $(3,0)$, $(0,3)$, $(-3,0)$ and $(0,-3)$.



Step 3

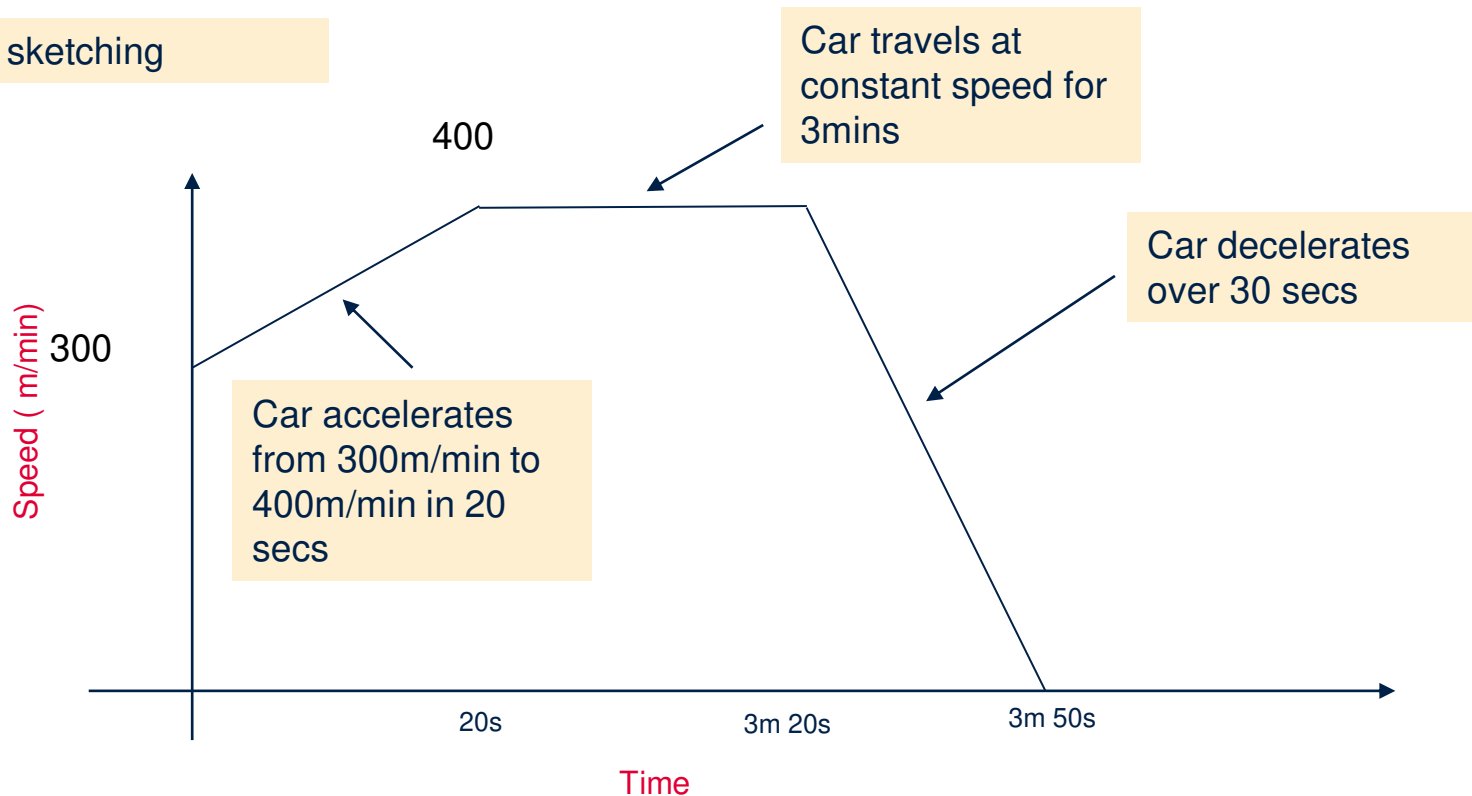
From the 2 sketches we can see that the graphs are separated at the two marked points and this will be the shortest distance, which is 4 units



A car is initially travelling at 300m/min, it speeds up over a 20 second interval with a constant acceleration to achieve a speed of 400 m/min. It travels at this speed for 3 minutes before slowing to a stop via constant de-acceleration over a period of 30 seconds.

Diagram not to scale

Start by sketching



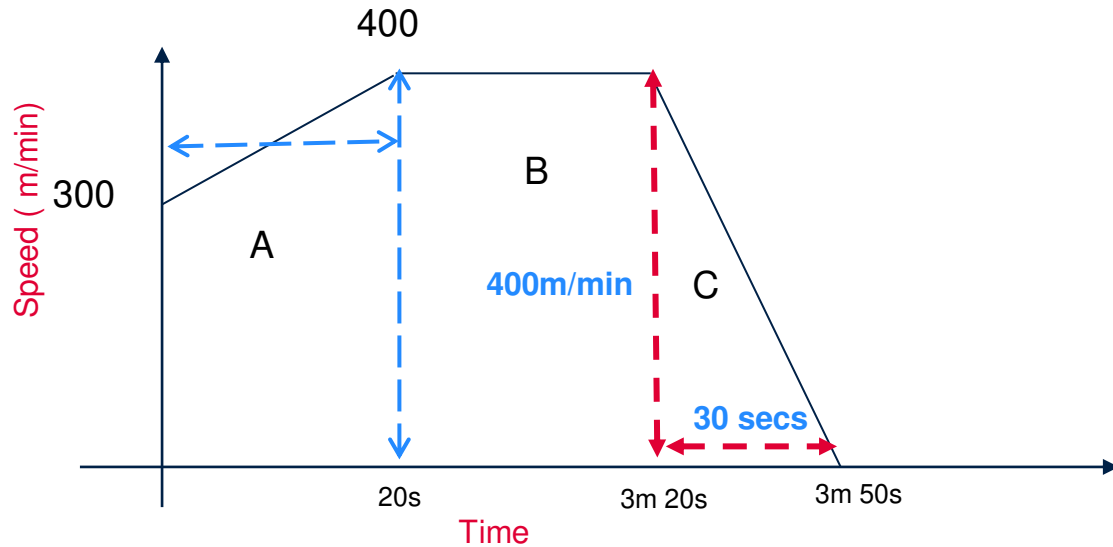


Diagram not to scale

Note: we need to be careful with units!

a) What is the cars' average speed for the first 20 seconds of travel?

Drawing the blue horizontal line on the graph is a visual way to show that the average speed is half way between 300m/min and 400m/min which is 350m/min

b) What is the cars' deceleration?

By drawing in the red lines we can see that it takes 30 secs for the car to stop - after travelling at 400m/min. (30 seconds = 0.5 minutes), so ...

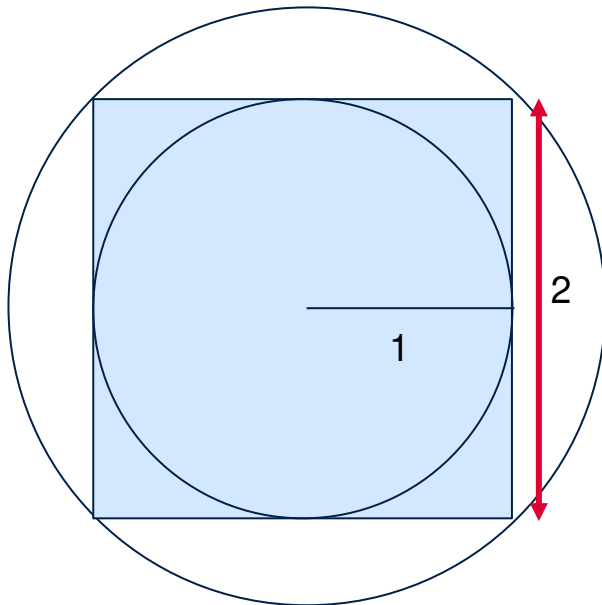
$$\text{Deceleration} = 400 \div 0.5 = 800\text{m/min}$$



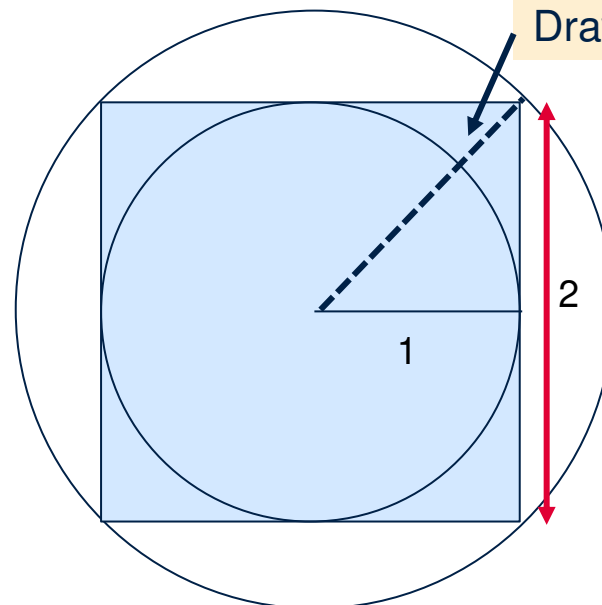
A square is placed inside a circle (C_1) so that the corners perfectly touch the circle's circumference.

Another circle (C_2) is now placed inside this square so that its circumference perfectly touches the square's sides.

What is the ratio of the lengths of the radius of C_1 and the radius of C_2 ?



Assume inner circle has radius of 1 unit. Therefore square has side length of 2.



Using Pythagoras' theorem we can calculate the length of radius of circle 1 to be $\sqrt{2}$.

Draw in radius of C_1

Therefore the ratio of radii of C_1 to C_2 is:

$$\sqrt{2} : 1$$



The activities on the next few slides may contain some content from A level maths; for that reason they are optional, but still fun and worth trying!



Solve $(\sin x + 1)(2\cos x - 1) = 0$ for $0 < x < 360^\circ$

A Triggly Problem



Solutions on the next slide....



$$\text{Solve } (\sin x + 1)(2\cos x - 1) = 0 \text{ for } 0 < x < 360^\circ$$

Fortunately, this is an already factorised quadratic. So.....

$$\sin x + 1 = 0$$

$$\sin x = -1$$

$$x = \sin^{-1}(-1)$$

$$x = 270^\circ$$

or

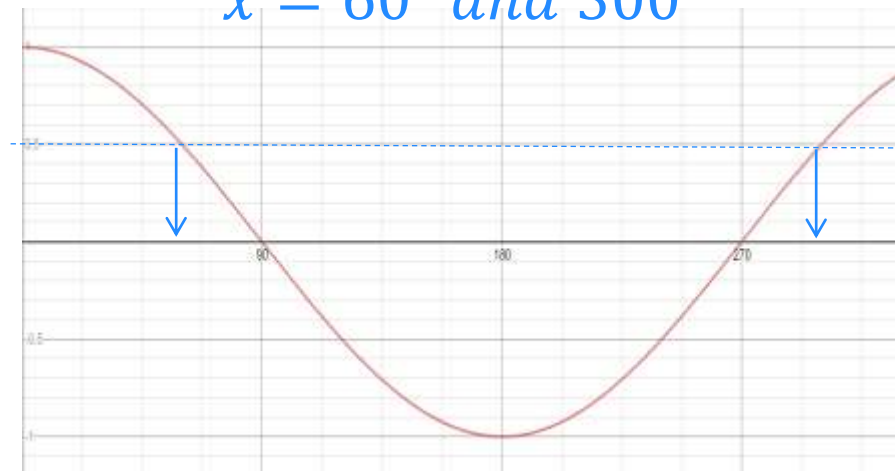
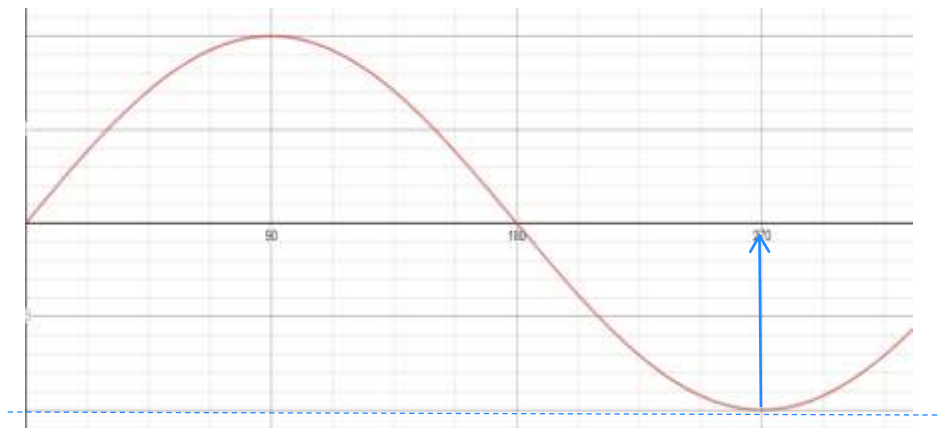
$$2\cos x - 1 = 0$$

$$2\cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \cos^{-1}\left(\frac{1}{2}\right)$$

$$x = 60^\circ \text{ and } 300^\circ$$





Which one of the equations below describes the graph?

- $y = (x + 1)(x - 1)(x - 2)$
- $y = -x(x - 1)(x + 1)$
- $y = x(x - 1)(x + 1)$



A cubic match up



Solutions on the next slides....



Which one of the equations below describes the graph?



Consider when $y = 0$ to find the x intercepts

- $$y = (x + 1)(x - 1)(x - 2)$$

$$x = -1, x = 1, x = 2$$

- $$y = -x(x - 1)(x + 1)$$

$$x = 0, x = 1, x = -1$$

- $$y = x(x - 1)(x + 1)$$

$$x = 0, x = 1, x = -1$$

Both of these equations have the correct intercepts – but which is the correct graph?

A negative cubic starts in the top left quadrant and finishes in the bottom right quadrant so it can't be the second equation

The correct equation is $y = x(x - 1)(x + 1)$



Extend what you have learnt about quadratics to help you match up the cubic graphs in this Desmos activity

You can join the activity without signing in or entering your real name.



Learn more about factorising cubics in this activity – with [solutions](#)

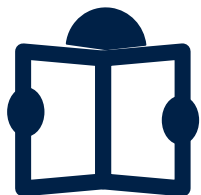


Click [here](#) to try our Exponential Marbleslides Challenge

You will be investigating the features of exponential graphs whilst trying to catch as many stars as possible



You can join the activity without signing in or entering your real name.



[Read](#) about Euclid's Axioms and discover how they might be used in this interactivity. Sketches and diagrams help with more than just questions about graphs!



[Play](#) 'Euclidea' to explore more about Euclidean Geometry and constructions.



[Watch](#) this video to see how you can 'graph' art! To see all the finalists in the Desmos Art competition (and get inspiration to enter it yourself in the future) click [here](#).

Contact the AMSP



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