



**Advanced Mathematics  
Support Programme®**

Substitute  $x = 9$  into the following two expressions

$$x^2 + 3x + 2$$

and

$$(x + 2)(x + 1)$$

What do you notice?

Substitute  $x = 9$  into the following two expressions

$$x^2 + 3x + 2$$

$$(9)^2 + 3(9) + 2 = 81 + 27 + 2 = 110$$

and

$$(x + 2)(x + 1)$$

$$(9 + 2)(9 + 1) = 11 \times 10 = 110$$

Both give the same answer as the expressions are equivalent

One of the expressions was a lot easier to evaluate! Why?

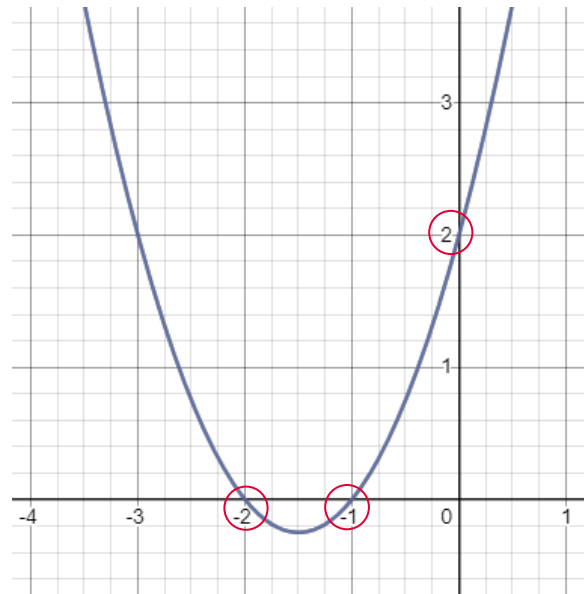
$$x^2 + 3x + 2 \text{ or } (x + 2)(x + 1)$$

expanded form

factorised form

$$y = x^2 + 3x + 2$$

$$y = (x + 2)(x + 1)$$



**Factorising** is a key skill for both sketching graphs and solving equations, both of which will be covered later.

Sometimes it is more helpful to factorise an expression, other times better to be expand it, depending on the context.



Factorise the following fully:

1.  $x^2 + 5x - 6$

5.  $k^2 - 2k - 24$

2.  $x^2 + 13x - 30$

6.  $p^2 - 10p + 21$

3.  $y^2 - 13y + 30$

7.  $x^2 - 16x$

4.  $t^2 + 2t - 15$

8.  $3x(2x - 1) + 4(1 - 2x)$



# Further Factorising 1



Solutions on the next slide....



$$1. \quad x^2 - 5x + 6 \quad \longrightarrow \quad = (x + 6)(x - 1)$$

$$2. \quad x^2 + 13x - 30 \quad \longrightarrow \quad = (x + 15)(x - 2)$$

$$3. \quad y^2 - 13y + 30 \quad \longrightarrow \quad = (y - 10)(y - 3)$$

$$4. \quad t^2 + 2t - 15 \quad \longrightarrow \quad = (t + 5)(t - 3)$$



$$5. \quad k^2 - 2k - 24 \quad \rightarrow \quad = (k - 6)(k + 4)$$

$$6. \quad p^2 - 10p + 21 \quad \rightarrow \quad = (p - 7)(p - 3)$$

$$7. \quad x^2 - 16x \quad \rightarrow \quad = x(x - 16)$$

$$8. \quad 3x(2x - 1) + 4(1 - 2x) \quad \rightarrow \quad = 3x(2x - 1) - 4(2x - 1)$$

Can you see  $-(2x - 1)$  is the same as  $(1 - 2x)$

Take  $-1$  out as a factor

$$= (2x - 1)(3x - 4)$$

The common factor to take out is  $(2x - 1)$





Factorise the following fully:

1.  $x^2 + 6x - 7$

5.  $k^2 + 9k + 20$

2.  $y^2 + y - 12$

6.  $x^2 + x - 56$

3.  $y^2 - 11y + 28$

7.  $p^2 - 25p$

4.  $t^2 - 7t - 18$

8.  $x^2(3x - 4) + (4 - 3x)$



## Further Factorising 2



Solutions on the next slide....



$$1. \quad x^2 + 6x - 7 \quad \longrightarrow \quad = (x + 7)(x - 1)$$

$$2. \quad y^2 + y - 12 \quad \longrightarrow \quad = (y + 4)(y - 3)$$

$$3. \quad y^2 - 11y + 28 \quad \longrightarrow \quad = (y - 7)(y - 4)$$

$$4. \quad t^2 - 7t - 18 \quad \longrightarrow \quad = (t - 9)(t + 2)$$



$$5. \quad k^2 + 9k + 20 \quad \longrightarrow \quad = (k + 5)(k + 4)$$

$$6. \quad x^2 + x - 56 \quad \longrightarrow \quad = (x + 8)(x - 7)$$

$$7. \quad p^2 - 25p \quad \longrightarrow \quad = p(p - 25)$$

Did you notice?  $-(3x - 4)$  is the same as  $(4 - 3x)$

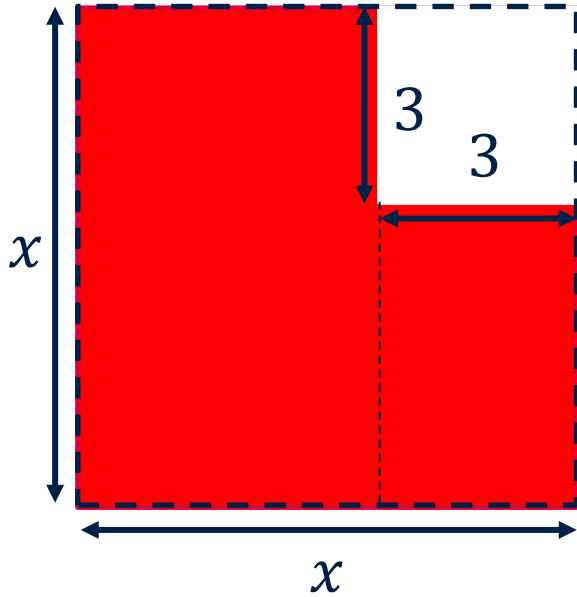
$$8. \quad x^2(3x - 4) + (4 - 3x) \quad \longrightarrow \quad = x^2(3x - 4) - (3x - 4)$$

The common factor to take out is  $(3x - 4)$

$$= (3x - 4)(x^2 - 1)$$

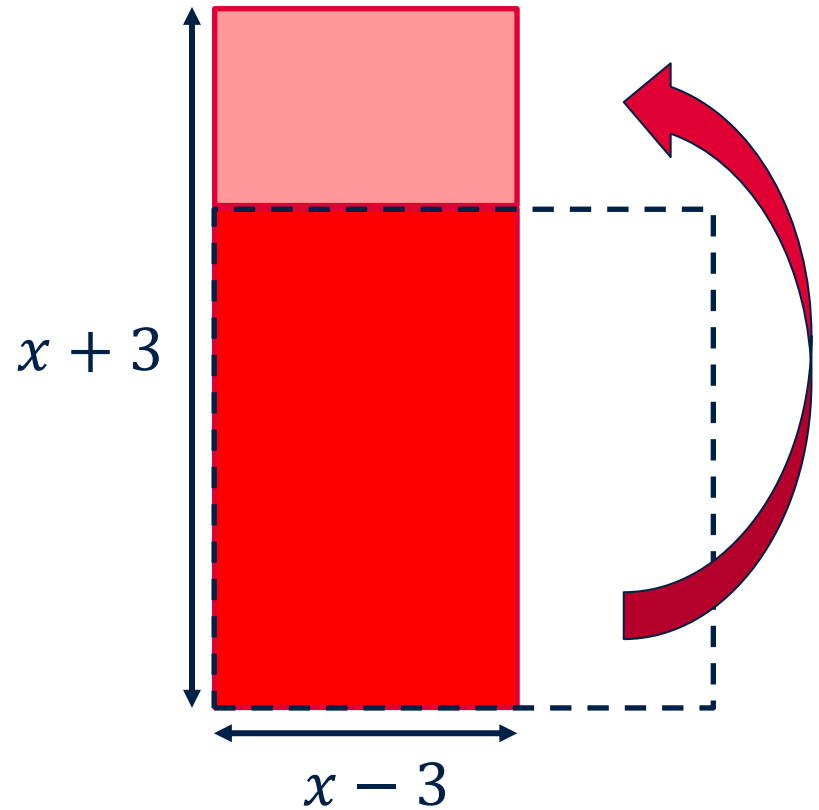


A special case for factorising is the **difference of two squares**. Expressions such as  $x^2 - 3^2$ , where the coefficient of  $x$  is zero.



$$x^2 - 3^2$$

=



$$(x - 3)(x + 3)$$



Try factorising these expressions using the difference of two squares

1.  $x^2 - 6^2$

2.  $y^2 - 144$

3.  $x^2 - y^2$

4.  $4t^2 - 81$

5.  $x^2 - 5$



Try factorising these expressions using the difference of two squares

$$1. \quad x^2 - 6^2 = (x - 6)(x + 6)$$

$$2. \quad y^2 - 144 = (y + 12)(y - 12)$$

$$3. \quad x^2 - y^2 = (x + y)(x - y)$$

$$4. \quad 4t^2 - 81 = (2t - 9)(2t + 9)$$

$$5. \quad x^2 - 5 = (x - \sqrt{5})(x + \sqrt{5})$$



So far we have been factorising quadratic expressions where  $a = 1$ . For example  $x^2 - 2x - 15$

Time to try some trickier quadratics!

Have a go at this one...

**Factorise**  
 $6x^2 + 19x + 10$





## Factorise


$$6x^2 + 19x + 10$$

- If you got  $6x^2 + 19x + 10 = (3x + 2)(2x + 5)$  Well done! ★

Feeling confident? You can skip on to the **Trickier Quadratics** questions.

There are many methods for factorising quadratics where  $a > 1$

- Follow this link to discover 'the grid method'.

Alternatively, if you want to refresh your memory on the method that you learnt at school - Search  **Tricky Quadratics** to find a video to help you.



Remember this from the **Expanding Double Brackets** section?

When using a grid we noticed the following:

$(3x - 5)$

$\times$	$3x$	$-5$
$x$	$3x^2$	$-5x$
$+3$	$+9x$	$-15$

$(x + 3)$

The **products** of the diagonals are identical expressions

$$9x \times -5x = -45x^2$$

$$3x^2 \times -15 = -45x^2$$

The **sum** of these terms make the middle term in the simplified expression

$$3x^2 + 9x - 5x - 15$$

$$3x^2 + 4x - 15$$

We are now going to use this method to help us factorise quadratics where the  $x^2$  coefficient is not 1



Let's start with our previous question.

**Factorise**  
 $6x^2 + 19x + 10$

We can put the  $6x^2$  and the  $+10$  straight into the grid as shown below

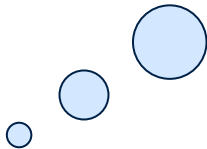
$\times$		
	$6x^2$	
		$+10$

The product of this diagonal is  $60x^2$

$\times$		
	$6x^2$	$?$
	$?$	$+10$

So the product of this diagonal is also  $60x^2$

Got some idea about what the missing terms might be already?  
 Remember their sum must also equal  $19x$





**Factorise**  
 $6x^2 + 19x + 10$

$\times$		
	$6x^2$	?
	?	$+10$

The product of this diagonal is  $60x^2$

The sum of this diagonal is  $19x$

Pairs of factors that make 60?....

.... that also sum to give 19?

$\times$		
	$6x^2$	$4x$
	$15x$	$+10$

It doesn't matter which order you put  $15x$  and  $4x$  into the grid as multiplication is commutative.



**Factorise**  
 $6x^2 + 19x + 10$

Time to factorise in the grid!

$\times$	?	?
	$6x^2$	$4x$
	$15x$	$+10$

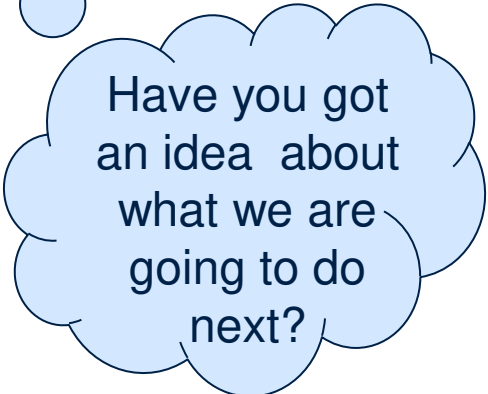
↑

↑

Find the Highest Common Factor (HCF) of each column and write it at the top

HCF of  $6x^2$  and  $15x$  is  $3x$   
 HCF of  $4x$  and  $10$  is  $2$

$\times$	$3x$	$2$
	$6x^2$	$4x$
	$15x$	$+10$





## Factorise

$$6x^2 + 19x + 10$$

Time to factorise the grid!

$\times$	$3x$	$2$
$?$	$6x^2$	$4x$
$?$	$15x$	$+10$

← Find the Highest Common Factor of each row and write them on the left

←

HCF of  $6x^2$  and  $4x$  is  $2x$   
 HCF of  $15x$  and  $10$  is  $5$

$\times$	$3x$	$2$
$2x$	$6x^2$	$4x$
$5$	$15x$	$+10$

This means that  $6x^2 + 19x + 10$  factorises to  $(2x + 5)(3x + 2)$



- Try factorising these expressions
- You might want to try the grid method.

1.  $3x^2 - 10x - 8$

2.  $2x^2 - 7x + 6$

3.  $4y^2 + 20y + 9$

4.  $6x^2 - 13x - 8$

5.  $20x^2 + x - 12$

\*Hint. There are some partially filled grids on the next slide if you want to use them



For some help with factorising you can complete the grids by filling in the blanks

×	$x$	
	$3x^2$	$-6x$
		$-8$

$$3x^2 - 10x - 8$$

×		
$2x$	$2x^2$	
	$-3x$	$+6$

$$2x^2 - 7x + 6$$

×		
	$4y^2$	
	$2y$	$+9$

$$4y^2 + 20y + 9$$

×		
		$-3x$
		$-8$

$$6x^2 - 13x - 8$$

×		
	$16x$	

$$20x^2 + x - 12$$





For some help with factorising you can complete the grids by filling in the blanks

×	$x$	$-2$
$3x$	$3x^2$	$-6x$
$4$	$4x$	$-8$

$$3x^2 - 10x - 8$$

$$= (3x + 4)(x - 2)$$

×	$x$	$2$
$2x$	$2x^2$	$-4x$
$3$	$-3x$	$+6$

$$2x^2 - 7x + 6$$

$$= (2x - 3)(x - 2)$$

×	$2y$	$9$
$2y$	$4y^2$	$18y$
$1$	$2y$	$+9$

$$4y^2 + 20y + 9$$

$$= (2y + 1)(2y + 9)$$

×	$3x$	$-8$
$2x$	$6x^2$	$-16x$
$1$	$3x$	$-8$

$$6x^2 - 13x - 8$$

$$= (2x + 1)(3x - 8)$$

×	$5x$	$4$
$4x$	$20x^2$	$16x$
$-3$	$-15x$	$-12$

$$20x^2 + x - 12$$

$$= (4x - 3)(5x + 4)$$



1.  $3x^2 - 10x - 8 = (3x + 2)(x - 4)$

2.  $2x^2 - 7x + 6 = (2x - 3)(x - 2)$

3.  $4y^2 + 20y + 9 = (2y + 1)(2y + 9)$

4.  $6x^2 - 13x - 8 = (3x - 8)(2x + 1)$

5.  $20x^2 + x - 12 = (5x + 4)(4x - 3)$



These expressions are slightly different to the previous ones, but can still be factorised.

1.  $2t^2 - 32$

2.  $x^3 - 7x^2 + 12x$

3.  $x^4 - x^2 - 2$

4.  $y^4 - 625$



These expressions are subtly different to the previous ones, but can still be factorised.

$$1. \quad 2t^2 - 32 = 2(t^2 - 16) = 2(t - 4)(t + 4)$$

$$2. \quad x^3 - 7x^2 + 12x = x(x^2 - 7x + 12) = x(x - 3)(x - 4)$$

$$3. \quad x^4 - x^2 - 2 = (x^2 - 2)(x^2 + 1)$$

$$4. \quad y^4 - 625 = \underbrace{(y^2 + 5)(y^2 - 5)}_{\text{Difference of two squares}} = \underbrace{(y^2 + 5)(y - 5)(y + 5)}_{\text{Difference of two squares - twice!}}$$

Difference of two squares – twice!



What is the value of each of the following?  
**calculators not allowed**

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

Hints available on the next slide



What is the value of each of the following?

$$9^2 - 1^2$$

$$99^2 - 1^2$$

$$999^2 - 1^2$$

- Can you factorise  $9^2 - 1^2$ ?
- How does this help?

# Without a calculator Solutions



Follow the [link](#) for the solutions



Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

Hints available on the next slide





Without using a calculator, find the value of

$$\frac{122 \times (122^2 + 4 \times 123)}{124} - \frac{124 \times (124^2 - 4 \times 123)}{122}$$

It might seem strange advice but.....

- Replace 123 by  $n$  and 122 by  $n-1$
- Now go on to factorise

# Still without a calculator Solutions



Follow the [link](#) for the solutions



Simplify

$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

Hints available on the next slide



Simplify

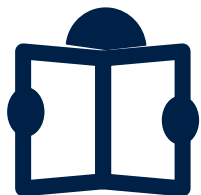
$$\frac{x^2 - 3x - 10}{x^2 + 7x + 10}$$

- Factorise the numerator then the denominator
- What do you notice?

# Top and Bottom Solution



Follow the [link](#) for the solutions



Explore the history of mathematics with this interactive historical timeline -in particular look for at Al-Khwarizmi. Can you find a famous artist and a mathematician whose triangle you met in the Expanding topic?



Discover how you can use factorising quadratics and apply it to higher powers by this neat trick shown in this rich task.



Watch how you can apply difference of two squares to a fun numerical problem.