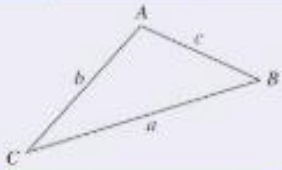
Topic B: Trigonometry 2

Bridging to Ch3.2

To solve problems involving non-right-angled triangles you use the **sine** rule and the **cosine** rule.



Key point

Sine rule: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ to find an angle,

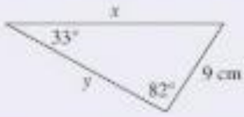
or write as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to find a side. It's important that sides and angles with the same letter are opposite.

Angles are written upper case and sides are lower case. Angle A is opposite side a and so on.

In order to use the sine rule you must have information about an opposite side and angle pair.

Example 1

Find the lengths of sides x and y in this triangle.



$$\frac{x}{\sin 82} = \frac{9}{\sin 33}$$

The side of length 9 cm is opposite the 33° angle. The side x is opposite the 82° angle. So you can use the sine rule.

$$x = \frac{9}{\sin 33} \times \sin 82 = 16.4 \text{ cm}$$

Rearrange to solve for x

The angle opposite y is $180 - 33 - 82 = 65°$

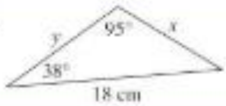
$$\text{So } \frac{y}{\sin 65} = \frac{9}{\sin 33}$$

$$y = \frac{9}{\sin 33} \times \sin 65 = 15.0 \text{ cm}$$

You could also use the other pair of x and 82°: $\frac{y}{\sin(65)} = \frac{16.4}{\sin(82)}$

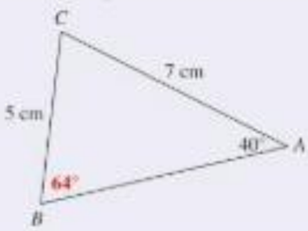
Try It 1

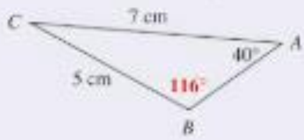
Find the lengths of sides x and y in this triangle.



When finding an angle, remember that the equation $\sin\theta = k$ has two solutions in the range $0° < \theta < 180°$ when $0 < k < 1$. So you need to subtract the first solution you find from 180° to find a second solution then decide whether or not it is a possible solution for your triangle.

For example, if $A = 40°$, $a = 5$ and $b = 7$ then two different triangles could be formed:



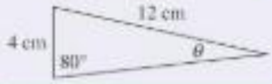


The sine rule gives the acute solution $B = 64°$, but $B = 180 - 64 = 116°$ is also a possible solution.

You need to check whether or not the obtuse solution will actually work. If it is too big then the angle sum of the triangle would be more than 180° which is not possible!

Example 2

Find the size of angle θ in this triangle.



$$\frac{\sin\theta}{4} = \frac{\sin 82}{12}$$

$$\sin\theta = \frac{\sin 80}{12} \times 4 = 0.328\ldots$$

$$\theta = \sin^{-1}(0.328\ldots) = 19.2°$$

$$\text{or } \theta = 180 - 19.2 = 160.8°$$

but 160.8° is not possible for this triangle as it would give an angle sum of more than 180°

So, θ = 19.2°

The side of length 12 cm is opposite the 80° angle so you can use the sine rule. The angle θ is opposite a side of length 4 cm.

Rearrange to solve for θ

Subtract from 180° to give other solution.

Try It 2

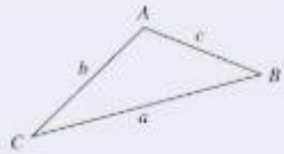
Find the size of angle θ in this triangle.

If you do not have information about an opposite side and angle pair then you will need to use the cosine rule.

Key point

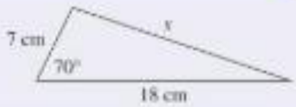
Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

(where angle A is opposite side a)



Example 3

Find the length of side x in this triangle.



$$x^2 = 7^2 + 18^2 - 2 \times 7 \times 18 \times \cos 70$$

$$= 286.8$$

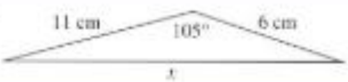
$$x = \sqrt{286.8} = 16.9 \text{ cm}$$

You do not have information about an opposite side and angle pair so you need to use the cosine rule.

x is the side you need to find so this is 'a' in the rule, which means the 70° angle is 'A' since it is opposite x. The other two sides are 'b' and 'c' in either order.

Try It 3

Find the length of side x in this triangle.



If you know the lengths of all three sides of a triangle then you can use the cosine rule to find one of the angles. You can either use $a^2 = b^2 + c^2 - 2bc\cos A$ and solve to find A, or you can use the rearranged version of the cosine rule:

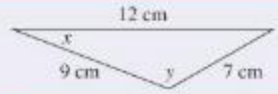
Key point

Cosine rule: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

(Remember that side a is opposite angle A)

Example 4

Find the size of angles x and y in this triangle.



$$\cos x = \frac{(12^2 + 9^2 - 7^2)}{2\times9\times12} = \frac{22}{27}$$

$$x = \cos^{-1}\left(\frac{22}{27}\right) = 35.4°$$

$$\cos y = \frac{9^2 + 7^2 - 12^2}{2\times9\times7} = -\frac{1}{9}$$

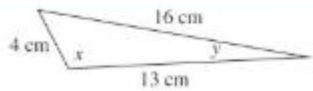
$$y = \cos^{-1}\left(-\frac{1}{9}\right) = 96.4°$$

You know the lengths of all three sides so use the cosine rule.

Make sure you use 12 as the side opposite to angle y

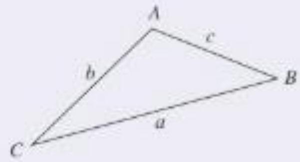
Try It 4

Find the size of angles x and y in this triangle.



The equation $\cos\theta = k$ only has one solution in the range $0° \le \theta \le 180°$ so the cosine rule gives a unique solution.

You can use trigonometry to find the area of a triangle.



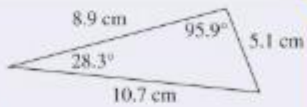
Key point

Area of triangle $= \frac{1}{2}ab\sin C$

where C is the angle between sides a and b

Example 5

Calculate the area of this triangle.



$$\text{Area} = \frac{1}{2}\times8.9\times10.7\times\sin28.3 = 22.6\text{ cm}^2$$

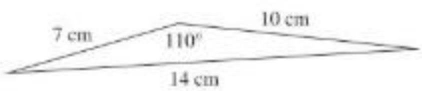
Or, using the 95.9° angle:

$$\text{Area} = \frac{1}{2}\times8.9\times5.1\times\sin95.9 = 22.6\text{ cm}^2$$

The 28.3° angle is between the 8.9 cm and 10.7 cm sides.

Try It 5

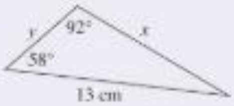
Calculate the area of this triangle



Bridging Exercise Topic B

Bridging to Ch3.2

1 Use the sine rule to find the length of the sides labelled x and y in each of these triangles.

a


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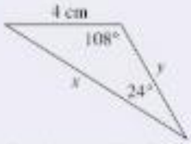
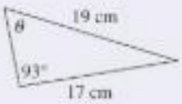


c


2 Use the sine rule to find the size of angle θ in each of these triangles. Explain whether the solution is unique in each case.

a


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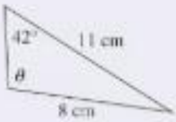

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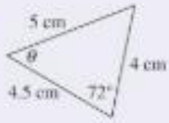

3 Use the cosine rule to find the length of the side x in each of these triangles.

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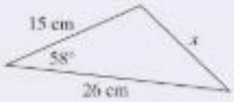

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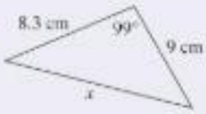

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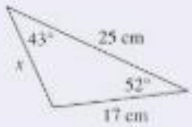

4 Use the cosine rule to find the size of the angles labelled x and y in each of these triangles.

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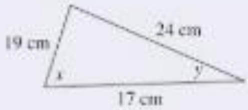

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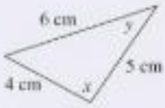

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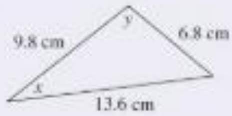
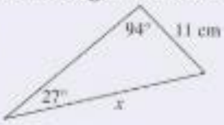


5 Find the length of the side labelled x in each of these triangles.

a


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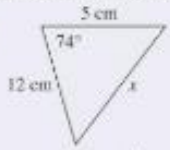

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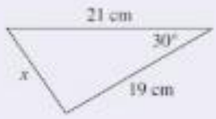

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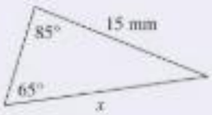

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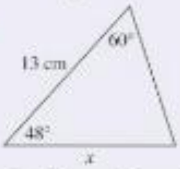

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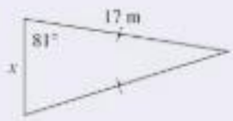

6 Find the size of the acute angle θ in each of these triangles.

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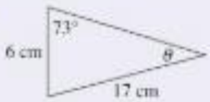

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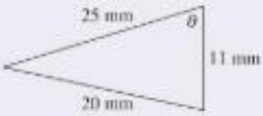

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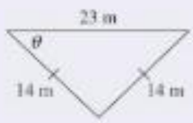

7 Triangle ABC is such that $AB = 5$ cm, $BC = 3$ cm and $AC = 7$ cm. Calculate the size of angle ABC

8 Triangle ABC is such that $AB = 24$ cm, $AC = 27$ cm, angle $ABC = 37°$ and angle $BCA = \theta$. Calculate θ

9 Find the area of each of the triangles in question **3**