Trigonometry A

Topic A: Trigonometry 1

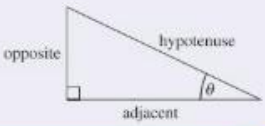
Bridging to Ch3.1

When dealing with right-angled triangles you can use the **sine, cosine and tangent ratios.**

Key point

For all right-angled triangles with angle θ:

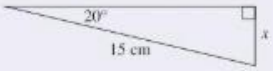
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}},\quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}},\quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



Example 1

Find the lengths of sides x and y in these right-angled triangles.

a



b

a $\sin 20 = \frac{x}{15}$

$x = 15\sin 20 = 5.13\text{ cm}$

b $\cos 75 = \frac{6}{y}$

$y = \frac{6}{\cos 75} = 23.2\text{ cm}$

Rearrange to make y the subject.

x is the opposite side to the 20° angle and you know that the hypotenuse is 15 cm, so use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

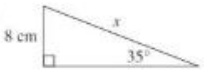
Rearrange to make x the subject.

y is the hypotenuse and you know that the adjacent side to the 75° angle is 6 cm, so use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Try It 1

Find the lengths of sides x and y in these right-angled triangles.

a



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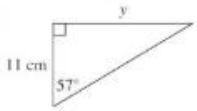


Example 2

Find the size of the angle marked x in the triangle.

$\cos x = \frac{5}{9}$

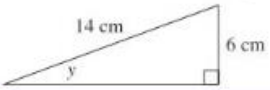
$x = \cos^{-1}\left(\frac{5}{9}\right) = 56.3°$

Find the inverse function $\cos^{-1}$ on your calculator.

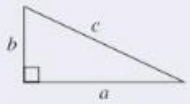
You know that the adjacent to the angle x is 5 cm and that the hypotenuse is 9 cm, so use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

Try It 2

Find the size of the angle marked y in the triangle.



Sometimes you may need to use **Pythagoras' theorem** along with the trigonometric ratios to solve problems.

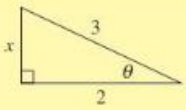


Key point

Pythagoras' theorem: $a^2+b^2=c^2$ for a right-angled triangle where c is the hypotenuse and a and b are the two shorter sides.

Example 3

Given that $\cos\theta = \frac{2}{3}$ for an acute angle θ, find the exact values of $\sin\theta$ and of $\tan\theta$



Since θ is an acute angle, you can solve this problem by drawing a right-angled triangle where $\cos\theta = \frac{2}{3}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ so label the side adjacent to θ as 2 and the hypotenuse as 3

$x^2+2^2=3^2 \Rightarrow x^2+4=9 \Rightarrow x^2=5 \Rightarrow x=\sqrt{5}$

Use Pythagoras' theorem to find the missing length, x

Therefore $\sin\theta = \frac{\sqrt{5}}{3}$

Since $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

and $\tan\theta = \frac{\sqrt{5}}{2}$

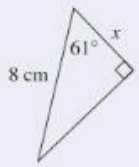
Since $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

Bridging Exercise Topic A

Bridging to Ch3.1

1 Find the side labelled x in each of these triangles.

a



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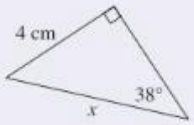

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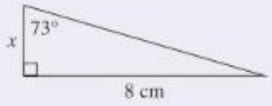

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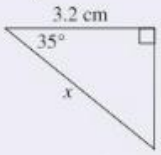

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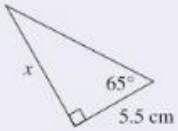

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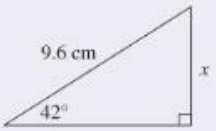

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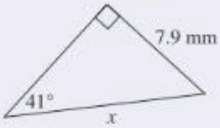

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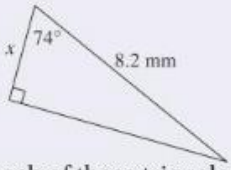

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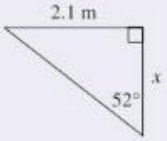
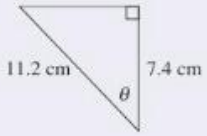


2 Find the angle labelled θ in each of these triangles.

a

b



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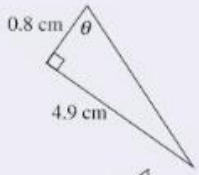

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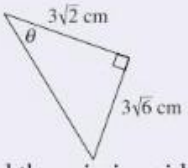

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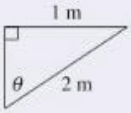

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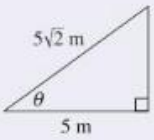
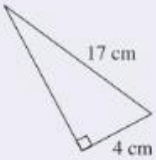


3 Find the missing side lengths in each of these triangles.

a



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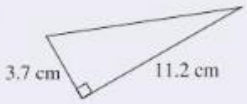

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