

Topic G: Circles



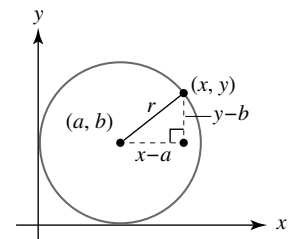
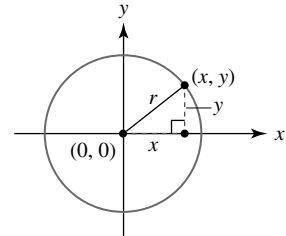
You can use the centre and radius of a circle to define its equation, and to define the equation of a tangent to circle at a given point. You can also find points of intersection between a circle and a line or chord.

Using Pythagoras' theorem, a circle of radius r , with centre at the origin, has equation $x^2 + y^2 = r^2$

Following a similar method, you can write down the equation of a circle with centre (a, b) and radius r , using a general point (x, y) on the circle, as shown in the diagram.

The horizontal distance between the centre (a, b) and the point on the circle (x, y) is the difference between the x -coordinates. The vertical distance between the centre (a, b) and the point on the circle (x, y) is the difference between the y -coordinates.

Using Pythagoras' theorem: $r^2 = (x-a)^2 + (y-b)^2$



A circle of radius r and centre (a, b) has equation $(x-a)^2 + (y-b)^2 = r^2$

Key point

Example 1

- a** Find the centre and radius of the circle with equation $(x-5)^2 + (y+1)^2 = 9$
- b** Write the equation of a circle with centre $(-3, 7)$ and radius 4

a The centre is at $(5, -1)$

The radius is $\sqrt{9} = 3$

b $a = -3, b = 7$ and $r = 4$

So equation is $(x+3)^2 + (y-7)^2 = 16$

Equation is $(x-5)^2 + (y-(-1))^2 = 9$ so $a = 5$ and $b = -1$

Remember to find the positive square root.

Remember to square the radius.

- a** Find the centre and radius of the circle with equation $(x+2)^2 + (y-8)^2 = 25$
- b** Write the equation of a circle with centre $(7, -9)$ and radius 8

Try It 1



You can use a diameter of a circle to find the equation of the circle.

If AB is the diameter of a circle then

Key point

- the centre of the circle is the midpoint of AB
- the radius of the circle is half the length of the diameter AB

Example 3

Find the equation of the circle with diameter AB where A is $(3, -8)$ and B is $(-5, 4)$

$$\begin{aligned} \text{Centre is } & \left(\frac{3+(-5)}{2}, \frac{(-8)+4}{2} \right) \\ & = (-1, -2) \end{aligned}$$

$$\begin{aligned} \text{Radius is } & \frac{1}{2} \sqrt{(-5-3)^2 + (4-(-8))^2} \\ & = \frac{1}{2} \sqrt{(-8)^2 + (12)^2} \\ & = 2\sqrt{13} \end{aligned}$$

$$\text{So the equation of the circle is } (x+1)^2 + (y+2)^2 = 52$$

The centre is the midpoint of AB . Use $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$

The radius is half of the length of AB

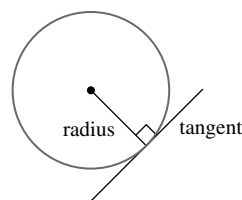
Use $(x-a)^2 + (y-b)^2 = r^2$ and remember to square the radius: $(2\sqrt{13})^2 = 52$

Find the equation of the circle with diameter AB where A is $(4, 6)$ and B is $(2, -4)$

Try It 3

A **tangent** to a circle is a line which is perpendicular to a radius of the circle. Note that a tangent will intersect a circle exactly once.

You can use these facts to find the equation of a tangent to a circle.



MyMaths



1152, 1953

SEARCH



Example 6

Show that $x - y = 12$ is a tangent to the circle $(x - 6)^2 + (y + 2)^2 = 8$

$$y = x - 12$$

$$(x - 6)^2 + (x - 12 + 2)^2 = 8$$

$$\Rightarrow (x - 6)^2 + (x - 10)^2 = 8$$

$$\Rightarrow x^2 - 12x + 36 + x^2 - 20x + 100 = 8$$

$$\Rightarrow 2x^2 - 32x + 128 = 0$$

$$b^2 - 4ac = (-32)^2 - 4 \times 2 \times 128 = 0$$

So they meet once only.
Hence $x - y = 12$ is a tangent to $(x - 6)^2 + (y + 2)^2 = 8$

- Rearrange the equation of the line to make either x or y the subject.
- Substitute for y (or x) into the equation of the circle.
- Expand the brackets.
- Simplify.
- If the discriminant is zero then there is exactly one solution.

To show that a line is a tangent to a circle you can show that they only intersect once.

Show that $2x - y + 11 = 0$ is a tangent to the circle $(x - 5)^2 + (y - 1)^2 = 80$

Try It 6

Three horizontal lines for writing, enclosed in a rounded rectangular border.



MyMaths



1152, 1953

SEARCH





1 Write the equations of these circles.

a circle with radius 7 and centre (2, 5)

b circle with radius 4 and centre (-1, -3)

c circle with radius $\sqrt{2}$ and centre (-3, 0)

d circle with radius $\sqrt{5}$ and centre (4, -2)

2 Find the centre and the radius of the circles with these equations.

a $(x-5)^2 + (y-3)^2 = 16$

b $(x+3)^2 + (y-4)^2 = 36$

c $(x-9)^2 + (y+2)^2 = 100$

d $(x+3)^2 + (y+1)^2 = 80$

e $(x-\sqrt{2})^2 + (y+2\sqrt{2})^2 = 32$

f $\left(x+\frac{1}{4}\right)^2 + \left(y+\frac{1}{3}\right)^2 = \frac{25}{4}$

3 Find the centre and the radius of the circles with these equations.

a $x^2 + 2x + y^2 = 24$

b $x^2 + y^2 + 12y = 13$

c $x^2 + y^2 - 4x + 3 = 0$

d $x^2 + y^2 + 6x + 8y + 2 = 0$

e $x^2 + y^2 - 8x - 10y = 3$

f $x^2 + y^2 + 14x - 2y = 5$

g $x^2 + y^2 + 5x - 4y + 3 = 0$

h $x^2 + y^2 - 3x - 9y = 2$

i $x^2 + y^2 - x + 7y + 12 = 0$

4 Find the equation of the circle with diameter AB where the coordinates of A and B are

a $(3, 5)$ and $(1, 7)$

b $(4, -1)$ and $(2, -5)$

c $(1, -3)$ and $(-9, -6)$

d $(-3, -7)$ and $(8, -16)$

e $(\sqrt{2}, 4)$ and $(-\sqrt{2}, 6)$

f $(4\sqrt{3}, -\sqrt{3})$ and $(-2\sqrt{3}, -5\sqrt{3})$

5 Determine whether each of these points lies on the circle with equation $(x-3)^2 + (y+2)^2 = 5$

a (5, 3)

b (1, -1)

c (4, 3)

d (2, 0)

6 Determine which of these circles the point $(-3, 2)$ lies on.

a $(x-5)^2 + y^2 = 68$

b $(x+2)^2 + (y+1)^2 = 8$

c $(x-6)^2 + (y-2)^2 = 81$

7 A circle has equation $(x-1)^2 + (y+1)^2 = 10$. Find the equation of the tangent to the circle through the point $(2, -4)$. Write your answer in the form $ax+by+c=0$ where a , b and c are integers.

- 8 A circle has equation $(x+3)^2 + (y+7)^2 = 34$. Find the equation of the tangent to the circle through the point $(0, -2)$. Write your answer in the form $ax+by+c=0$ where a, b and c are integers.

- 9 A circle has equation $x^2 + (y-8)^2 = 153$. Find the equation of the tangent to the circle through the point $(3, -4)$. Write your answer in the form $y=mx+c$

- 10 A circle has equation $(x+4)^2 + y^2 = 20.5$. Find the equation of the tangent to the circle through the point $(0.5, -0.5)$. Write your answer in the form $y=mx+c$

11 Find the points of intersection, A and B , between these pairs of lines and circles.

a $x + y = 5, x^2 + y^2 = 53$

b Calculate the length of the chord AB

14 Show that the line $y = x - 3$ is a tangent to the circle $(x - 3)^2 + (y + 2)^2 = 2$

15 Show that the line $4x + y = 34$ is a tangent to the circle $(x + 1)^2 + (y - 4)^2 = 68$

16 Show that the line $x + 3y = 25$ is a tangent to the circle $x^2 + (y - 5)^2 = 10$

17 Show that the line $y = 2x + 3$ does not intersect the circle $(x - 1)^2 + (y + 4)^2 = 1$

18 Show that the line $3x + 4y + 2 = 0$ does not intersect the circle $(x + 3)^2 + (y - 6)^2 = 9$
