1 Bridging Unit: Algebra 1

Topic E: The quadratic formula

Bridging to Ch1.4

You can solve a quadratic equation using the **quadratic formula**. The quadratic formula can also be used to quickly determine how many roots a quadratic equation has.

Key point

The quadratic formula for $ax^2+bx+c=0$ is $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Example 1

Solve the equation $3x^2-5x-7=0$ using the quadratic formula.

$a=3, b=-5, c=-7$

Substitute into the formula, taking care with negatives.

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4\times3\times(-7)}}{2\times3}$$

$$=\frac{5\pm\sqrt{109}}{6}$$

$=2.57$ or -0.91 (to 2 dp)

Use your calculator to give answer as a decimal:

$\dfrac{5+\sqrt{109}}{6}=2.57$ and

$\dfrac{5-\sqrt{109}}{6}=-0.91$

You can also use the equation solver on your calculator to solve quadratic equations.

Try It 1

Use the quadratic formula to solve the quadratic equation $7x^2-4x-6=0$

Inside the square root of the quadratic formula you have the expression b^2-4ac. This expression is called the **discriminant**. You can use the discriminant to determine how many roots the equation has.

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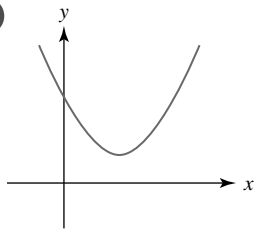
Key point

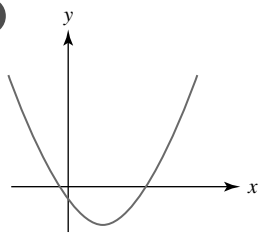
1. If $b^2 - 4ac < 0$ then the equation has no real roots.
2. If $b^2 - 4ac > 0$ then the equation has two real roots.
3. If $b^2 - 4ac = 0$ then the equation has one real root.

1



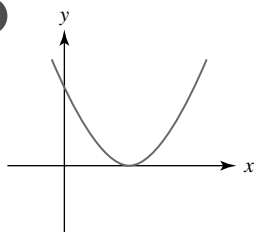
The curve does not cross the x-axis so the discriminant is negative.

2



The curve crosses the x-axis twice so the discriminant is positive.

3



The curve touches the x-axis once so the discriminant equals zero.

Example 2

Given that the quadratic equation $x^2 + 3x + k + 1 = 0$ has exactly one solution, find the value of k

$a = 1, b = 3, c = k + 1$

So $b^2 - 4ac = 3^2 - 4 \times 1 \times (k + 1)$ — Find the discriminant.

$= 5 - 4k$

$5 - 4k = 0 \Rightarrow k = \frac{5}{4}$ — The equation has exactly one solution so the discriminant is zero.

Try It 2

Given that the quadratic equation $kx^2 - x + 5 = 0$ has exactly one solution, find the value of k

Example 3

Given that the quadratic equation $5x^2+3x-k=0$ has real solutions, find the range of possible values of k

$a=5, b=3, c=-k$

So $b^2-4ac=5^2-4\times5\times(-k)$ — Find the discriminant.

$=25+20k$

$25+20k\geq0 \Rightarrow k\geq-\frac{5}{4}$ — The equation has real solutions so the discriminant is greater than or equal to zero.

Try It 3

Given that the quadratic equation $x^2+3x-k=0$ has real solutions, find the range of possible values of k

Example 4

Given that the quadratic equation $-x^2+7x+3-k=0$ has no real solutions, find the range of possible values of k

$a=-1, b=7, c=3-k$

So $b^2-4ac=7^2-4\times(-1)\times(3-k)$ — Find the discriminant.

$=61-4k$

$61-4k<0 \Rightarrow k>\frac{61}{4}$ — The equation has no solutions so the discriminant is negative.

Try It 4

Given that the quadratic equation $kx^2-7x+1=0$ has no real solutions, find the range of possible values of k

Bridging Exercise Topic E

Bridging to Ch1.4

1 Use the quadratic formula to solve each of these equations.

a $7x^2+3x-8=0$

b $-x^2+4x-2=0$

c $x^2-12x+4=0$

2 Work out how many real solutions each of these quadratic equations has.

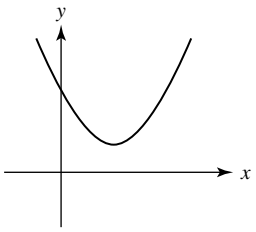
a $x^2-5x+7=0$

b $7-2x-3x^2=0$

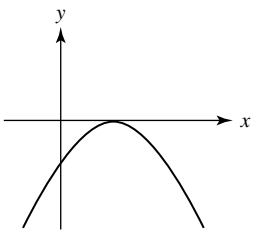
c $4x^2-28x+49=0$

3 Choose a possible equation from the box for each of the graphs.

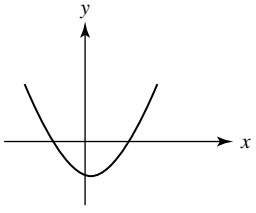
a



b



c



$y=-4x^2+12x-9$

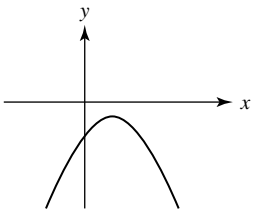
$y=-x^2+2x-4$

$y=7x^2-5x+4$

$y=-x^2+x+6$

$y=6x^2-x-15$

d



4 Find the value of k in each equation given that they each have exactly one solution.

a $3x^2+2x-k=0$

b $kx^2-x+4=0$

c $2x^2+5x+k-5=0$

5 Find the range of possible values of k for each equation given that they all have real solutions.

a $x^2+3x-3k=0$

b $kx^2-7x+4=0$

c $-x^2+6x-k-2=0$

6 Find the range of possible values of k for each equation given that they all have no real solutions.

a $5x^2 - x + 2k = 0$

b $-kx^2 + 4x + 5 = 0$

c $6x^2 - 5x + 3 - 2k = 0$