1 Bridging Unit: Algebra 1

Topic D: Completing the square

Bridging to Ch1.4

Some quadratics are **perfect squares** such as $x^2-8x+16$ which can be written $(x-4)^2$. For other quadratics you can **complete the square**. This means write the quadratic in the form $(x+q)^2+r$

Key point

The completed square form of x^2+bx+c is $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$

If you have an expression of the form ax^2+bx+c then first factor out the a, as shown in Example 1

Example 1

Write each of these quadratics in the form $p(x+q)^2+r$ where *p*, *q* and *r* are constants to be found.

a x^2+6x+7 **b** $-2x^2+12x$

a $x^2+6x+7=\left(x+\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2+7$

$=(x+3)^2-9+7$

$=(x+3)^2-2$

The constant term in the bracket will be half of the coefficient of *x*

b $-2x^2+12x=-2\left[x^2-6x\right]$

$=-2\left[(x-3)^2-9\right]$

$=-2(x-3)^2+18$

First factor out the coefficient of x^2 then complete the square for the expression in the square brackets.

Try It 1

Write each of these quadratics in the form $p(x+q)^2+r$

a x^2+22x **b** $2x^2-8x-6$ **c** $-x^2+10x$

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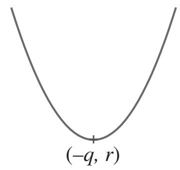
Key point

The turning point on the curve with equation $y = p(x+q)^2 + r$ has coordinates $(-q, r)$, this will be a minimum if p is positive and a maximum if p is negative.



Example 2

Find the coordinates of the turning point of the curve with equation $y = -x^2 + 5x - 2$

$$
\begin{aligned}
-x^2 + 5x - 2 &= -\left[x^2 - 5x + 2\right] \\
&= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2\right] \\
&= -\left[\left(x - \frac{5}{2}\right)^2 - \frac{17}{4}\right] \\
&= -\left(x - \frac{5}{2}\right)^2 + \frac{17}{4}
\end{aligned}
$$

So the maximum point is at $\left(\frac{5}{2}, \frac{17}{4}\right)$

First factor out the –1 then complete the square for the expression in the square brackets.

The curve is at its highest point when the bracket is equal to zero: $x - \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2}$

Try It 2

Find the coordinates of the turning point of each of these curves and state whether they are a maximum or a minimum.

a $y = x^2 - 3x + 1$ **b** $y = -x^2 - 7x - 12$ **c** $y = 2x^2 + 4x - 1$

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Bridging Exercise Topic D

1 Write each of these quadratic expressions in the form $p(x+q)^2+r$

a x^2+8x

b x^2-18x

c x^2+6x+3

d $x^2+12x-5$

e $x^2-7x+10$

f x^2+5x+9

g $2x^2+8x+4$

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h $3x^2+18x-6$

i $2x^2-10x+3$

j $-x^2+12x-1$

k $-x^2+9x-3$

l $-2x^2+5x-1$

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2 Use completing the square to find the turning point of each of these curves and state whether it is a maximum or a minimum.

a $y = x^2 + 14x$

b $y = x^2 - 18x + 3$

c $y = x^2 - 9x$

d $y = -x^2 + 4x$

e $y = x^2 + 11x + 30$

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f $y=-x^2+6x-7$

g $y=2x^2+16x-5$

h $y=-3x^2+15x-2$

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