

## Topic C: Factorising quadratics and simple cubics



Expressions such as  $5x^2 + x$ ,  $2x^2 + 4$  and  $x^2 + 2x - 1$  are called **quadratics** and can sometimes be factorised into two linear factors. There are three types of quadratics to consider:

- 1 Quadratics of the form  $ax^2 + bx$  have a common factor of  $x$  so can be factorised using a single bracket and removing the highest common factor of the two terms, e.g.  $6x^2 + 8x = 2x(3x + 4)$
- 2 Quadratics of the form  $x^2 + bx + c$  will sometimes factorise into two sets of brackets. You need to find two constants with a product of  $c$  and a sum of  $b$ , e.g.  
 $x^2 - 3x + 2 = (x - 2)(x - 1)$  since  $-2 \times -1 = 2$  and  $-2 + -1 = -3$
- 3 Quadratics of the form  $ax^2 - c$  will factorise if  $a$  and  $c$  are square numbers. This is called the **difference of two squares**, e.g.  $4x^2 - 9 = (2x + 3)(2x - 3)$

### Example 1

Factorise each of these quadratics.

**a**  $9x^2 + 15x$

**b**  $x^2 + 3x - 10$

**c**  $x^2 - 16$

**a**  $9x^2 + 15x = 3x(3x + 5)$

**b**  $x^2 + 3x - 10 = (x + 5)(x - 2)$

**c**  $x^2 - 16 = (x + 4)(x - 4)$

The highest common factor of  $9x^2$  and  $15x$  is  $3x$

You need to find two constants with a product of  $-10$  and a sum of  $3$ :  $5 \times -2 = -10$  and  $5 + -2 = 3$  so the constants are  $-2$  and  $5$

$x^2$  and  $16$  are both square numbers.

Factorise each of these quadratics.

**a**  $14x^2 - 7x$

**b**  $x^2 - 5x + 4$

**c**  $x^2 - 25$

Try It 1

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**Example 3**

Use factorisation to find the roots of these quadratic equations.

**a**  $4x^2 + 12x = 0$

**b**  $5x^2 = 21x - 4$

**a**  $4x^2 + 12x = 4x(x + 3)$

$4x(x + 3) = 0 \Rightarrow 4x = 0$  or  $x + 3 = 0$

If  $4x = 0$  then  $x = 0$  and if  $x + 3 = 0$  then  $x = -3$

**b**  $5x^2 - 21x + 4 = 0$

$5x^2 - 21x + 4 = 5x^2 - 20x - x + 4$

$= 5x(x - 4) - (x - 4)$

$= (5x - 1)(x - 4)$

$(5x - 1)(x - 4) = 0 \Rightarrow 5x - 1 = 0$  or  $x - 4 = 0$

If  $5x - 1 = 0$  then  $x = \frac{1}{5}$  and if  $x - 4 = 0$  then  $x = 4$

Factorise the quadratic.

One of the factors must be equal to zero.

Solve to find the roots.

Rearrange so you have a quadratic expression equal to zero.

Write  $-21x = -x - 20x$  since  $-20 \times -1 = 20$  and  $5 \times 4 = 20$

Factorise the quadratic.

The product is zero so one of the factors must be equal to zero.

Solve to find the roots.

Find the roots of these quadratic equations.

**Try It 3**

**a**  $6x^2 - 12x = 0$

**b**  $4x^2 = 23x - 15$

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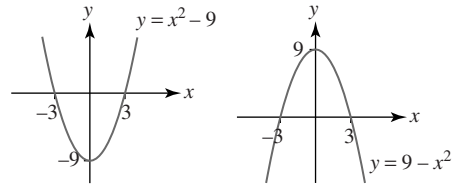
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A quadratic function has a **parabola** shaped curve.

When you sketch the graph of a quadratic function you must include the coordinates of the points where the curve crosses the  $x$  and  $y$  axes.



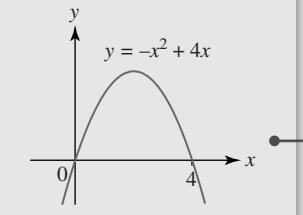
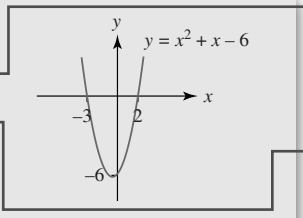
**Example 4**

Sketch these quadratic functions.

**a**  $y = x^2 + x - 6$       **b**  $y = -x^2 + 4x$

**a** When  $x = 0$ ,  $y = -6$   
 When  $y = 0$ ,  $x^2 + x - 6 = 0$   
 $x^2 + x - 6 = (x + 3)(x - 2)$   
 $(x + 3)(x - 2) = 0 \Rightarrow x = -3$  or  $x = 2$

**b** When  $x = 0$ ,  $y = 0$   
 When  $y = 0$ ,  $-x^2 + 4x = 0$   
 $-x^2 + 4x = -x(x - 4)$   
 $-x(x - 4) = 0 \Rightarrow x = 0$  or  $x = 4$



Find the  $y$ -intercept by letting  $x = 0$

Find the  $x$ -intercept by letting  $y = 0$

Factorise to find the roots.

Sketch the parabola and label the  $y$ -intercept of  $-6$  and the  $x$ -intercepts of  $-3$  and  $2$

Sketch the parabola, it will be this way up since the  $x^2$  term in the quadratic is negative. Label the  $x$  and  $y$  intercepts.

Factorise to find the roots.

Find the  $y$ -intercept by letting  $x = 0$

Find the  $x$ -intercept by letting  $y = 0$

Sketch these quadratic functions.

**Try It 4**

**a**  $y = x^2 - 25$

**b**  $y = x^2 + 10x + 25$

**c**  $y = 5x - x^2$



1 Fully factorise each of these quadratics.

a  $3x^2 + 5x$  \_\_\_\_\_

b  $8x^2 - 4x$  \_\_\_\_\_

c  $17x^2 + 34x$  \_\_\_\_\_

d  $18x^2 - 24x$  \_\_\_\_\_

2 Factorise each of these quadratics.

a  $x^2 + 5x + 6$  \_\_\_\_\_

b  $x^2 - 7x + 10$  \_\_\_\_\_

c  $x^2 - 5x - 6$  \_\_\_\_\_

d  $x^2 + 3x - 28$  \_\_\_\_\_

e  $x^2 - x - 72$  \_\_\_\_\_

f  $x^2 + 2x - 48$  \_\_\_\_\_

**g**  $x^2 - 12x + 11$  \_\_\_\_\_  
\_\_\_\_\_

**h**  $x^2 - 5x - 24$  \_\_\_\_\_  
\_\_\_\_\_

**3** Factorise each of these quadratics.

**a**  $x^2 - 100$  \_\_\_\_\_  
\_\_\_\_\_

**b**  $x^2 - 81$  \_\_\_\_\_  
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**c**  $4x^2 - 9$  \_\_\_\_\_  
\_\_\_\_\_

**d**  $64 - 9x^2$  \_\_\_\_\_  
\_\_\_\_\_

**4** Factorise each of these quadratics.

**a**  $3x^2 + 7x + 2$  \_\_\_\_\_  
\_\_\_\_\_

**b**  $6x^2 + 17x + 12$  \_\_\_\_\_  
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**c**  $4x^2 - 13x + 3$

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**d**  $2x^2 - 7x - 15$

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**e**  $2x^2 + 3x - 5$

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**f**  $7x^2 + 25x - 12$

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**g**  $8x^2 - 22x + 15$

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**h**  $12x^2 + 17x - 5$

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5 Fully factorise each of these quadratics.

a  $16x^2 - 25$

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b  $4x^2 - 16x$

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c  $x^2 + 13x + 12$

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d  $3x^2 + 16x - 35$

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e  $x^2 + x - 12$

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f  $100 - 9x^2$

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g  $2x^2 - 14x$

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h  $20x^2 - 3x - 2$

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**6** Use factorisation to find the roots of these quadratic equations.

**a**  $21x^2 - 7x = 0$

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**b**  $x^2 - 36 = 0$

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**c**  $17x^2 + 34x = 0$

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**d**  $6x^2 + 13x + 5 = 0$

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**e**  $4x^2 - 49 = 0$

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**f**  $x^2 = 7x + 18$

**g**  $x^2 - 7x + 6 = 0$

**h**  $21x^2 = 2 - x$

**i**  $17x = 5x^2 + 6$

**j**  $16x^2 + 24x + 9 = 0$

**k**  $9x^2 + 4 = 12x$

**1**  $40x^2 + x = 6$

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**7** Sketch each of these quadratic functions, labelling where they cross the  $x$  and  $y$  axes.

**a**  $y = x(x-3)$

**b**  $y = -x(3x+2)$

**c**  $y = x(3 - x)$

**d**  $y = (x + 2)(x - 2)$

**e**  $y = (x + 4)^2$

**f**  $y = -(2x+5)^2$

**g**  $y = (x-5)(x+2)$

**h**  $y = (x+1)(5-x)$

**8** Sketch each of these quadratic functions, labelling where they cross the  $x$  and  $y$  axes.

**a**  $y = x^2 + 6x$

**b**  $y = 3x^2 - 12x$

**c**  $y = x^2 - 121$



**d**  $y = x^2 - 3x - 10$

**e**  $y = -x^2 + 3x$

**f**  $y = 15x - 10x^2$

**g**  $y = 49 - x^2$

**h**  $y = -x^2 + 2x + 3$

**i**  $y = x^2 - 4x + 4$

**j**  $y = -x^2 + 14x - 49$

**k**  $y = 3x^2 + 4x + 1$

**l**  $y = -2x^2 + 11x - 12$