

Topic B: Solving linear equations and rearranging formulae

 Bridging
to Ch1.4

This topic recaps the **balance** method to solve problems involving linear equations, and both the **elimination** and **substitution** methods to solve linear simultaneous equations. You can solve linear equations and inequalities using the **balance** method where the same operation is applied to both sides.

Example 1

Solve the equation $7x - 5 = 3x - 2$

$$4x - 5 = -2$$

$$4x = 3$$

$$x = \frac{3}{4}$$

Divide both sides of the equation by 4

Subtract $3x$ from both sides of the equation.

Add 5 to both sides of the equation.

Solve the equation $3x + 8 = 5x - 6$

Try It 1

Example 2

Solve the inequality $5(x - 2) \leq 2x + 1$

$$5x - 10 \leq 2x + 1$$

$$3x - 10 \leq 1$$

$$3x \leq 11$$

$$x \leq \frac{11}{3}$$

First expand the brackets.

Subtract $2x$ from both sides.

Add 10 to both sides.

Divide both sides by 3

Solve the inequality $7x - 4 > x + 8$

Try It 2

When solving inequalities, remember that multiplying or dividing by a negative number will reverse the inequality sign. For example, $5 > 3$ but $-5 < -3$

Equations and formulae can be rearranged using the same method as for solving equations.

Example 3

Rearrange $Ax - 3 = \frac{x+B}{2}$ to make x the subject.

$$2Ax - 6 = x + B$$

$$2Ax - 6 - x = B$$

$$2Ax - x = B + 6$$

$$x(2A - 1) = B + 6$$

$$x = \frac{B + 6}{2A - 1}$$

Multiply both sides by 2

Subtract x from both sides.

Add 6 to both sides.

Divide both sides by $(2A - 1)$ to make x the subject.

Factorise the side involving x

Rearrange $3(x + A) = Bx + 1$ to make x the subject.

Try It 3

You can solve linear simultaneous equations using the **elimination** method, as shown in Example 4. The solutions to simultaneous equations give the point of intersection between the lines represented by the two equations.

Example 4

Solve the simultaneous equations $5x - 4y = 17$, $3x + 8y = 5$

$$\begin{aligned}
 15x + 40y &= 25 \quad (1) \\
 15x - 12y &= 51 \quad (2) \\
 (1) - (2): 52y &= -26 \\
 y &= -\frac{1}{2} \\
 5x - 4\left(-\frac{1}{2}\right) &= 17 \\
 5x + 2 &= 17 \\
 5x &= 15 \\
 x &= 3
 \end{aligned}$$

Multiply the second equation by 5

Multiply the first equation by 3

Subtract equation (2) from equation (1) to eliminate x

Solve this equation to find the value of x

Substitute $y = -\frac{1}{2}$ into one of the original equations.

Solve the simultaneous equations $2x + 5y = 1$, $3x - 2y = -27$

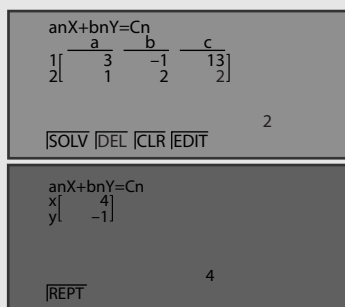
Try It 4

Calculator



Try it on your calculator

You can use a calculator to solve linear simultaneous equations.



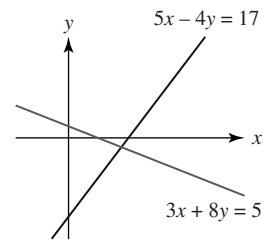
Activity

Find out how to solve the simultaneous equations $3x - y = 13$ and $x + 2y = 2$ on your calculator.



The example shows you that the lines $5x - 4y = 17$ and $3x + 8y = 5$ intersect at the point $\left(3, -\frac{1}{2}\right)$

If you are given the equation of two lines where y is the subject then the easiest way to solve these simultaneously is to use the **substitution** method as shown in the next example.



Example 5

Find the point of intersection between the lines with equations $y = 2x + 5$ and $y = 7 - 3x$

$$2x + 5 = 7 - 3x$$

$$5x + 5 = 7$$

$$5x = 2$$

$$x = 0.4$$

$$y = 2(0.4) + 5$$

$$= 5.8$$

So the lines intersect at the point $(0.4, 5.8)$

Substitute $2x + 5$ for y in the second equation.

Solve to find the value of x

Substitute $x = 0.4$ into either of the original equations to find the y -coordinate.

Find the point of intersection between the lines $y = 3x + 4$ and $y = 6x - 2$

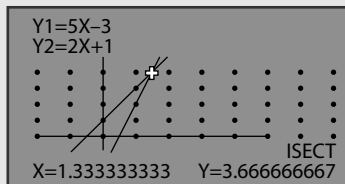
Try It 5

Calculator



Try it on your calculator

You can use a graphics calculator to find the point of intersection of two lines.



Activity

Find the point of intersection of the lines $y = 5x - 3$ and $y = 2x + 1$ on your graphics calculator.





1 Solve each of these linear equations.

a $3(2x+9)=7$

b $7-3x=12$

c $\frac{x+4}{5}=7$

d $2x+7=5x-6$

e $8x-3=2(3x+1)$

f $\frac{2x+9}{12} = x-1$

g $2(3x-7) = 4x$

h $7-2x = 3(4-5x)$

2 Solve each of these linear inequalities.

a $\frac{x}{2} + 7 \geq 5$

b $3-4x < 15$

c $5(x-1) > 12+x$

d $\frac{x+1}{3} > 2$

e $8x-1 \leq 2x-5$

f $3(x+1) \geq \frac{x-3}{2}$

g $3(2x-5) < 1-x$

h $x - (3 + 2x) \geq 2(x + 1)$

3 Rearrange each of these formulae to make x the subject.

a $2x + 5 = 3A - 1$

b $x + u = vx + 3$

c $\frac{3x - 1}{k} = 2x$

d $5(x-3m)=2nx-4$

e $(1-3x)^2=t$

f $\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$

g $\frac{1}{x^2+k}-6=4$

h $\sqrt{x+A} = 2B$

4 Use algebra to solve each of these pairs of simultaneous equations.

a $5x+12y = -6, \quad x+5y = 4$

b $7x+5y = 14, \quad 3x+4y = 19$

c $2x - 5y = 4$, $3x - 8y = 5$

d $3x - 2y = 2$, $8x + 3y = 4.5$

e $5x - 2y = 11$, $-2x + 3y = 22$

f $8x+5y=-0.5, -6x+4y=-3.5$

5 Use algebra to find the point of intersection between each pair of lines.

a $y=8-3x, y=2-5x$

b $y = 7x - 4, y = 3x - 2$

c $y = 2x + 3, y = 5 - x$

d $y + 5 = 3x, y = -5x + 7$

e $y = \frac{1}{2}x + 3, y = 5 - 2x$

f $y = 3(x + 2), y = 7 - 2x$
