
# 1 Bridging Unit: Algebra 1

## Topic A: Indices and surds

Bridging to Ch1.2, 1.3

You can apply the rules of indices and surds to simplify algebraic expressions. The following expressions can be simplified in **index form**:

**Key point**

$x^a \times x^b = x^{a+b}$ $\quad$ $x^a \div x^b = x^{a-b}$ $\quad$ $(x^a)^b = x^{ab}$

**Example 1**

Simplify these expressions. **a** $2x^3 \times 3x^5$ **b** $12x^7 \div 4x^3$ **c** $(3x^5)^3$

**a** $2x^3 \times 3x^5 = 6x^{3+5}$

$= 6x^8$

Multiply the coefficients together and use $x^a \times x^b = x^{a+b}$

**b** $12x^7 \div 4x^6 = \dfrac{12x^7}{4x^6}$

$= 3x$

Since $\dfrac{12}{4} = 3$ and $x^a \div x^b = x^{a-b}$ so $\dfrac{x^7}{x^6} = x^1$ which we just write as $x$

**c** $(3x^5)^3 = 3^3(x^5)^3$

Both the 3 and the $x^5$ must be raised to the power 3

$= 27x^{15}$

Since $(x^a)^b = x^{ab}$

**Try It 1**

Simplify these expressions.

**a** $5x^3 \times 2x^7$ **b** $18x^9 \div 3x^2$ **c** $(2x^6)^4$ **d** $\left(\dfrac{x^3}{3}\right)^2$

MyMaths 1033, 1045, 1064, 1065, 1951 SEARCH



 Acknowledgements: www.oxfordsecondary.co.uk/acknowledgements

Roots can also be expressed using indices, such that the square root of $x$ is written as $\sqrt{x} = x^{\frac{1}{2}}$

In general:

**Key point**

The $n$th root of $x$ is written $\sqrt[n]{x} = x^{\frac{1}{n}}$, and this can be raised to a power to give $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

**Key point**

A power of −1 indicates a reciprocal, so $x^{-1} = \frac{1}{x}$ and, in general, $x^{-n} = \frac{1}{x^n}$

**Example 2**

Evaluate each of these without using a calculator.

**a** $25^{0.5}$ **b** $6^{-2}$ **c** $8^{\frac{2}{3}}$

**a** $25^{0.5} = 25^{\frac{1}{2}}$

$= \sqrt{25}$ — Since a power of $\frac{1}{2}$ represents a square root.

$= 5$

**b** $6^{-2} = (6^2)^{-1}$

$= \frac{1}{6^2}$ — Since a power of −1 represents a reciprocal.

$= \frac{1}{36}$

**c** $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$ — Always calculate a root before a power.

$= 2^2$ — Since the cube root of 8 is 2

$= 4$

**Try It 2**

Evaluate each of these without a calculator.

**a** $36^{\frac{1}{2}}$ **b** $27^{\frac{2}{3}}$ **c** $64^{-0.5}$ **d** $\left(\frac{1}{2}\right)^4$

MyMaths 1033, 1045, 1064, 1065, 1951 SEARCH

## Example 3

Write these expressions in simplified index form.

**a** $\sqrt[3]{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{2x}{\sqrt{x}}$

**a** $\sqrt[3]{x} = x^{\frac{1}{3}}$

**b** $\frac{2}{x^3} = 2x^{-3}$

**c** $\frac{2x}{\sqrt{x}} = \frac{2x}{x^{\frac{1}{2}}}$ — Since $\sqrt{x} = x^{\frac{1}{2}}$

$= 2x^{1-\frac{1}{2}}$ — Subtract the powers, remembering that $x = x^1$

$= 2x^{\frac{1}{2}}$

## Try It 3

Write these expressions in simplified index form.

**a** $\sqrt[5]{x^2}$ **b** $\frac{3}{\sqrt{x}}$ **c** $\frac{3x^2}{\sqrt{x}}$ **d** $\frac{\sqrt{x}}{3x}$

MyMaths 1033, 1045, 1064, 1065, 1951 SEARCH



 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

A **surd** is an irrational number involving a root, for example $\sqrt{2}$ or $\sqrt[3]{7}$.
You can multiply and divide surds using the rules:

**Key point**

$$\sqrt{a}\times\sqrt{b}=\sqrt{ab} \text{ and } \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$

An irrational number is a real number that cannot be written as a fraction $\frac{a}{b}$, where $a$ and $b$ are integers with $b\neq 0$

You can simplify surds by finding square-number factors, for example $\sqrt{12}=\sqrt{4}\sqrt{3}=2\sqrt{3}$. It may also be possible to simplify expressions involving surds by collecting like terms or by **rationalising the denominator**. Rationalising the denominator means rearranging the expression to remove any roots from the denominator.

**Key point**

To rationalise the denominator, multiply both the numerator and denominator by a suitable expression:

$\frac{1}{\sqrt{a}}\ \frac{\sqrt{a}}{\sqrt{a}}=\frac{\sqrt{a}}{a}$, (multiply numerator and denominator by $\sqrt{a}$)

$\frac{1}{a+\sqrt{b}}\times\frac{a-\sqrt{b}}{a-\sqrt{b}}=\frac{a-\sqrt{b}}{a^2-b}$, (multiply numerator and denominator by $a-\sqrt{b}$)

$\frac{1}{a-\sqrt{b}}\times\frac{a+\sqrt{b}}{a+\sqrt{b}}=\frac{a+\sqrt{b}}{a^2-b}$, (multiply numerator and denominator by $a+\sqrt{b}$)

## Example 4

Simplify these expressions without using a calculator.

**a** $\sqrt{18}+5\sqrt{2}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{2}{1-\sqrt{5}}$

**a** $\sqrt{18}=\sqrt{9}\sqrt{2}$ — 9 is a square-number factor of 18 so you can simplify $\sqrt{18}$

$=3\sqrt{2}$

Therefore $\sqrt{18}+5\sqrt{2}=3\sqrt{2}+5\sqrt{2}$

$=8\sqrt{2}$ — Collect like terms.

**b** $\frac{6}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}$ — Rationalise the denominator by multiplying numerator and denominator by $\sqrt{3}$

$=\frac{6\sqrt{3}}{3}$

$=2\sqrt{3}$ — Since $6\div 3=2$

**c** $\frac{2}{1-\sqrt{5}}=\frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$ — Rationalise the denominator by multiplying numerator and denominator by $1+\sqrt{5}$

$=\frac{2(1+\sqrt{5})}{-4}$ — $(1-\sqrt{5})(1+\sqrt{5})=1-\sqrt{5}+\sqrt{5}-5$
$=1-5=-4$

$=-\frac{1}{2}(1+\sqrt{5})$



 Acknowledgements: www.oxfordsecondary.co.uk/acknowledgements

Try It 4

Simplify these expressions without using a calculator.

**a** $3\sqrt{28}-\sqrt{7}$ **b** $\frac{4}{\sqrt{3}}$ **c** $\frac{3}{1+\sqrt{2}}$ **d** $\frac{\sqrt{5}}{\sqrt{5}-2}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

# Bridging Exercise Topic A

Bridging to Ch1.2, 1.3

**1** Evaluate each of these without using a calculator.

**a** $49^{\frac{1}{2}}$

**b** $27^{\frac{1}{3}}$

**c** $5^{-1}$

**d** $64^{-\frac{1}{3}}$

**e** $9^{\frac{3}{2}}$

**f** $16^{\frac{3}{4}}$

**g** $125^{-\frac{2}{3}}$

**h** $\left(\frac{1}{2}\right)^{3}$

**i** $\left(\frac{1}{9}\right)^{-2}$

**j** $\left(\frac{4}{9}\right)^{\frac{1}{2}}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**k** $\left(\frac{9}{16}\right)^{-0.5}$

**l** $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

**2** Simplify these expressions fully without using a calculator.

**a** $\sqrt{8}$

**b** $\sqrt{75}$

**c** $2\sqrt{24}$

**d** $3\sqrt{48}$

**e** $\sqrt{20}+\sqrt{5}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**f** $\sqrt{27}-\sqrt{12}$

**g** $5\sqrt{32}-3\sqrt{8}$

**h** $\sqrt{50}+3\sqrt{125}$

**i** $\sqrt{68}+3\sqrt{17}$

**j** $3\sqrt{72}-\sqrt{32}$

**k** $4\sqrt{18}-2\sqrt{3}$



Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**l** $6\sqrt{5}+\sqrt{50}$

**3** Simplify these expressions fully without using a calculator.

**a** $\dfrac{1}{\sqrt{7}}$

**b** $\dfrac{2}{\sqrt{8}}$

**c** $\dfrac{12}{\sqrt{3}}$

**d** $\dfrac{\sqrt{8}}{\sqrt{12}}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**e** $\frac{1}{1+\sqrt{3}}$

**f** $\frac{2}{1+\sqrt{2}}$

**g** $\frac{8}{1-\sqrt{5}}$

**h** $\frac{2}{\sqrt{5}-1}$

**i** $\frac{\sqrt{2}}{2+\sqrt{3}}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**j** $\frac{2\sqrt{3}}{\sqrt{6}-2}$

**k** $\frac{1+\sqrt{2}}{1-\sqrt{2}}$

**l** $\frac{3+\sqrt{5}}{\sqrt{5}-3}$

**4** Expand the brackets and fully simplify each expression.

**a** $(1+\sqrt{2})(3+\sqrt{2})$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**b** $(1+\sqrt{2})(3-\sqrt{2})$

**c** $(1-\sqrt{2})(3+\sqrt{2})$

**d** $(1-\sqrt{2})(3-\sqrt{2})$

**e** $(\sqrt{3}+2)(4+\sqrt{3})$

**f** $(\sqrt{3}+2)(4-\sqrt{3})$

**g** $(\sqrt{3}-2)(4+\sqrt{3})$

**h** $(\sqrt{3}-2)(4-\sqrt{3})$

**i** $(\sqrt{6}+1)(\sqrt{2}+3)$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**j** $(\sqrt{6}+1)(\sqrt{2}-3)$

**k** $(\sqrt{6}-1)(\sqrt{2}+3)$

**l** $(\sqrt{6}-1)(\sqrt{2}-3)$

**5** Write each of these expressions in simplified index form.

**a** $x^3 \times x^7$

**b** $7x^5 \times 3x^6$

**c** $5x^4 \times 8x^7$

**d** $x^8 \div x^2$

**e** $8x^7 \div 2x^9$

**f** $3x^8 \div 12x^7$

**g** $(x^5)^7$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**h** $(x^2)^{-5}$

**i** $(3x^2)^4$

**j** $(6x^5)^2$

**k** $\sqrt{x^3}$

**l** $\sqrt[4]{x^5}$

**m** $\frac{5\sqrt{x}}{x}$

**n** $2x\sqrt{x}$

**o** $\frac{x^2}{3\sqrt{x}}$

**p** $x^3(x^5-1)$

**q** $x^3(\sqrt{x}+2)$

**r** $\dfrac{x+2}{x^3}$

**s** $\dfrac{\sqrt{x}+3}{x}$

**t** $\dfrac{(3-x^3)}{\sqrt{x}}$

**u** $(\sqrt{x}+3)^2$

**v** $\dfrac{3+\sqrt{x}}{x^2}$

**w** $\dfrac{1-x}{2\sqrt{x}}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**

**x** $\dfrac{\sqrt{x}+2}{3x^3}$


 Acknowledgements: **www.oxfordsecondary.co.uk/acknowledgements**