Maths tutorial booklet for

M3: Graphs

Name:____

Target grade:____

<u>Quiz scores:</u>

M3.1 Translate information between graphical, numerical and algebraic forms =

M3.2 Plot two variables from experimental or other data =

M3.3 Understanding that $y = mx + c$ represents a linear relationship =

M3.4 Determine the intercept of a graph =

M3.5 Calculate rate of change from a graph showing a linear relationship =

M3.6 Draw and use the slope of a tangent to a curve as a measure of rate of change =

University of East Anglia

M3.1 – Translate information between graphical, numerical and algebraic forms

Tutorials

Learners may be tested on their ability to:

- understand that data may be presented in a number of formats and be able to use these data, e.g. dissociation curves.

Translating information

You need to be able to represent numerical data as a graph, and you need to be able to do the reverse: read numerical data from a graph.

Translate from numerical to graphical – drawing a graph.

Representing numerical data as a graph simply means 'drawing graphs'. You've been doing this in science and maths lessons for years. Refer to M1.3 for a reminder of the different types of graphs and when to use them. The OCR Practical Skills Handbook also has useful information about how to present data in graphical form. The key point always is to think about what you are trying to communicate and draw your graph in the way that makes this clearest to whoever will be seeing it.

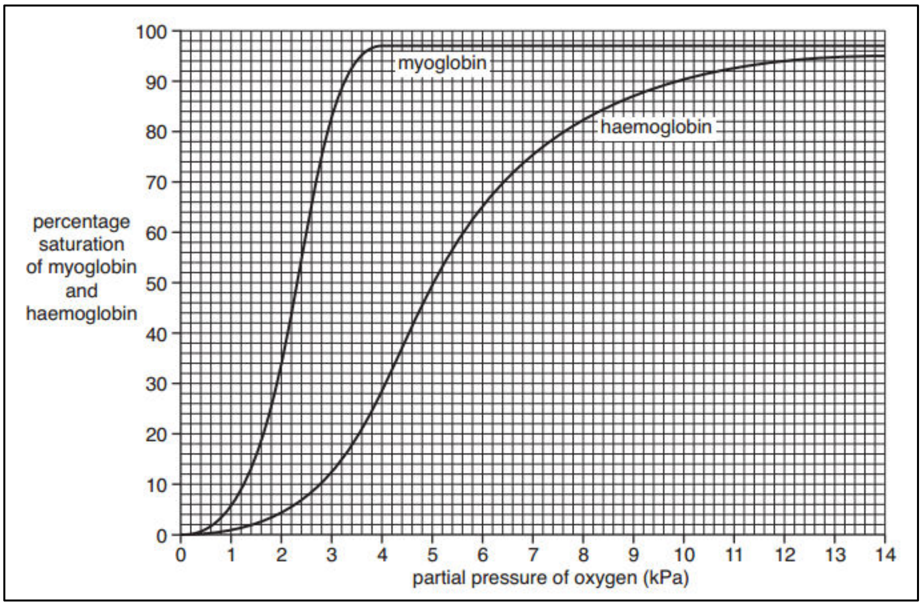
Translate from graphical to numerical – reading data from a graph.

When looking at a graph you need to be able to pick any value along the x axis (or any bar if it is a bar chart) and accurately identify the corresponding value on the y axis. And the other way round – for a value on the y axis you should be able to identify and report the corresponding value (or values – there could be more than one!) on the x axis. This is a simple-seeming skill but it takes care to get it right every time. It's all too easy to make 'silly' mistakes by misreading the scale.

When describing graphical data you need to make sure you refer to both the x axis and y axis variables, using co-ordinates if possible. In addition you need to accurately describe the relationship between the variables using data from the graph.

- Refer to x and y axis variables
- Use coordinates if possible
- Describe the relationship between the variables

For example, here are oxygen dissociation curves for myoglobin and haemoglobin.



You could be asked to find the percentage saturation of oxygen at a particular partial pressure, compare oxygen dissociation curves for different proteins or species, or to consider how the dissociation curve changes at different concentrations of carbon dioxide (the Bohr effect).

For example, when describing the features of the myoglobin curve you could describe the steep gradient in the middle part of the curve by saying:

“At pO_2 between 1 and 3kPa the percentage saturation of haemoglobin increases rapidly as pO_2 increases”.

This statement clearly refers to both the x axis and y axis variables and accurately describes the relationship using data from the graph.

Translate from graphical to algebraic and back.

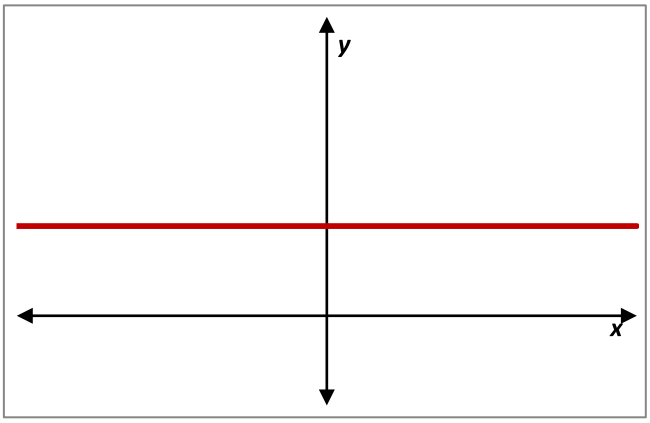
You also need to be able to identify some relationships between variables and either use graphs to show the relationship, or describe the relationship when you are shown it as a graph.

This will become a lot clearer when we look at a couple of examples but first let’s look at the (short!) list of relationships you need to deal with.

1. Nothing is changing

A graph showing a horizontal straight line parallel to the x-axis shows that the variable plotted on the y-axis is independent of the variable plotted on the x-axis. Or, in other words, the value of the variable y is always the same (constant) no matter what the value of the variable x.

$$y = constant$$

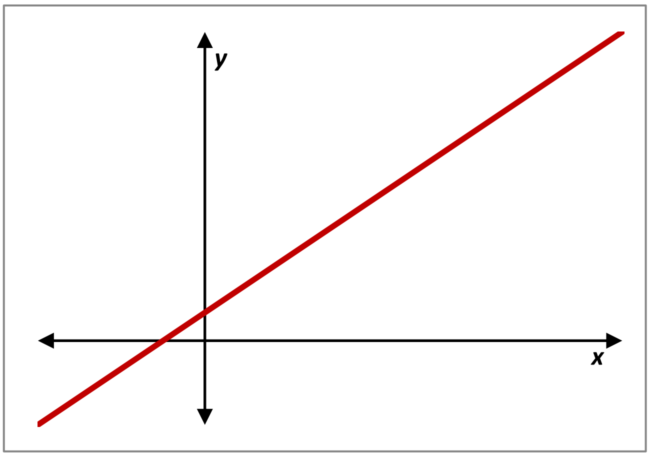


2. Linear growing (or shrinking!)

A second kind of graph that you need to know is a non-horizontal straight line. This graph shows that the variable plotted on the y-axis is linearly proportional to the variable plotted on the x-axis, or y α x. This can also be expressed as y = mx + c, where m is the gradient of the graph and c is the value of the intercept on the y axis. This sort of graph is covered in more detail in M3.3.

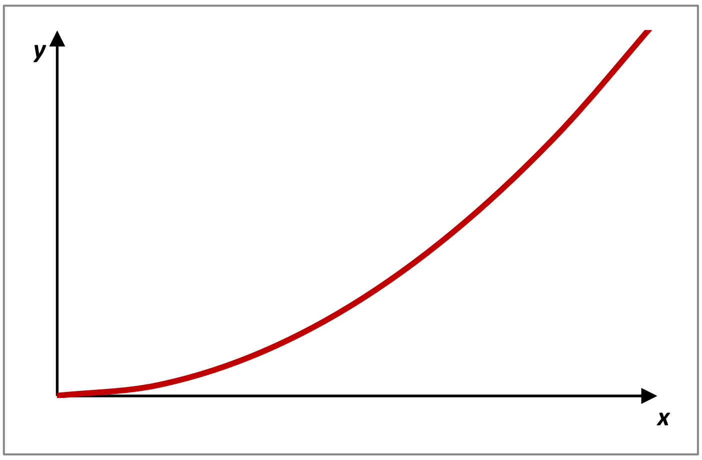
$$y \propto x$$

$$y = \mathrm{m}x + c$$



3. Non-linear growing (or shrinking!)

A third kind of graph is one where there is some dependence of the y variable on the x variable but it is not a linear relationship. The y variable might increase according to the square or the cube of the x variable (resulting in a steeper and steeper curve on the graph).



You do not need to be able to identify and distinguish these different kinds of non-linearity except for the case of exponential growth. As long as you know it is a non-linear relationship you have done enough.

Refer to M2.5 to see an example of exponential growth in the context of the growth of microorganisms and to see how a logarithmic scale allows you to turn this type of graph into a linear representation.

Exponential is a special kind of non-linear relationship, where the y variable changes according to some number raised to the power of the x variable.

$$y = n^x$$

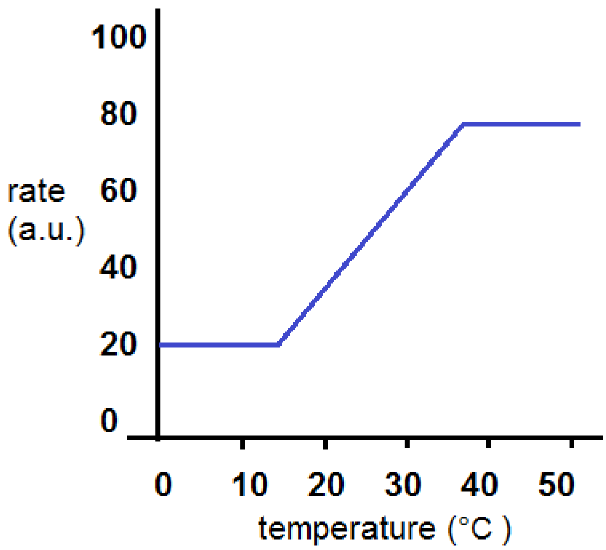
Note that the word ‘exponential’ gets used in everyday speech in quite an imprecise way, often simply to mean ‘getting bigger quickly’. In Biology we should try to only describe the relationship we see as ‘exponential’ when we believe, or suspect, that our y variable really is changing as n^x.

Examples

In the first graph below we can see that the relationship between the x and y variables (temperature and rate respectively) is different in different sections of the graph.

At temperatures below 15°C and above 37°C the rate is unaffected by changes in temperature. Rate is independent of temperature.

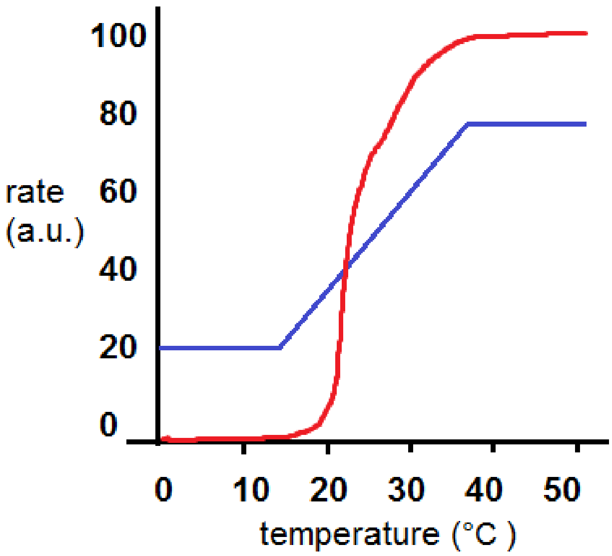
Between 15°C and 37°C the relationship between the variables is linear. Rate increases in direct proportion to the increase in temperature.



Now let's have a look at a second curve (shown in red) added to the original graph.

It shows a similar independent relationship between rate and temperature below 15°C and above 37°C but the relationship between these temperatures is clearly non-linear.

We would not describe the relationship as exponential between 15°C and 37°C but we might suspect that there is a small section between about 15°C and 20°C where rate could be increasing exponentially as temperature rises.

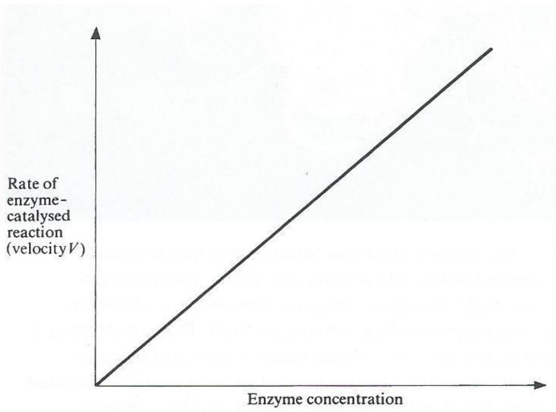


M3.1 – Translate information between graphical, numerical and algebraic forms

Quiz

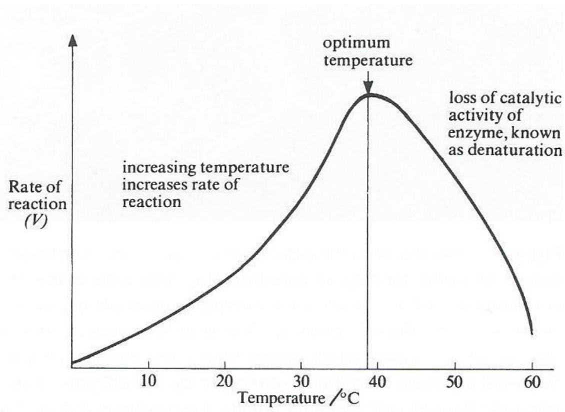
1. Describe the graphs below focussing on the relationship between the x and y axis variables.

A.



Relationship between enzyme concentration and the rate of an enzyme-controlled reaction.

B.



Effect of temperature on the rate of an enzyme-controlled reaction.

2. A simplified description of photosynthesis: 'Photosynthesis is dependent on light. When there is no light no photosynthesis takes place. As light intensity increases, the rate of photosynthesis increases linearly until it reaches an upper limit. Further increases in light intensity beyond this point have no effect on rate of photosynthesis.'

Draw a sketch graph to show this description of the relationship between rate of photosynthesis and light intensity.

How would the curve you have drawn change if you were to represent the following modifications to the description?

a) at very low light intensity no photosynthesis occurs – a threshold light intensity must be reached before any photosynthesis happens

b) in addition, at very high light intensity the chlorophyll is damaged and the rate of photosynthesis drops sharply

M3.2 – Plot two variables from experimental or other data

Tutorials

Learners may be tested on their ability to:

- select an appropriate format for presenting data, e.g. bar charts, histograms, line graphs and scattergrams.

Plotting variables

The best format for presenting data depends on what data you have.

Bar charts and histograms

Refer to M1.3 for a reminder about when to use a bar chart or a histogram.

Here is the summary table from M1.3 to help decide whether to use a bar chart or histogram and as a reminder of the differences when plotting:

Bar chart	Histogram
Qualitative data (categoric or rankable) Discrete quantitative data	Continuous quantitative data
Bars the same width	Differing widths of bars possible but not advised
Bars not touching	Bars touch

Line graphs and scattergrams

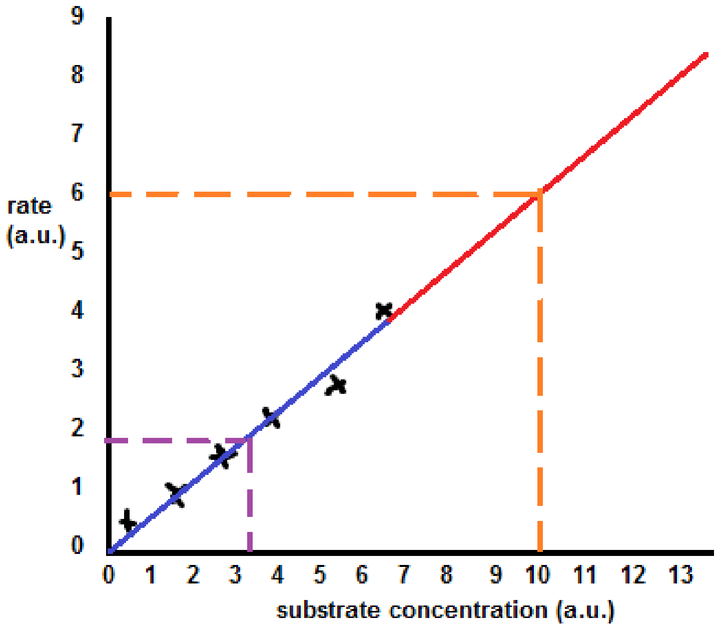
Line graphs and scattergrams can be used when we have data where each data point has two variables.

In many experimental situations we are able to identify one of these variables as ‘independent’ (generally this will be the variable that we change) and the other as ‘dependent’ (generally this is the variable that we measure – to see what effect changing the independent variable has had).

Line graphs

For example, we might carry out an enzymatic reaction using different substrate concentrations to see what effect this has on the rate at which product is made. The substrate concentration is the independent variable and the rate of product formation is the dependent variable.

These kinds of experimental data are best presented as a line graph. We plot the data, with the independent variable on the x axis and the dependent on the y axis. If we can identify a trend we add a line of best fit (straight or curved). We can use this line of best fit to interpolate (which simply means read off values in between our data points) and, in some cases, extrapolate (which means going beyond the range of our data points to read off other values).



In the line graph above the data points are shown as black crosses. The line of best fit has been drawn, including extrapolation beyond the range of substrate concentrations used. For clarity the extrapolated portion has been shown in red while the part of the line confined to the data range is in blue. When plotting your graphs and adding a line of best fit, use black for the whole line. Extrapolate only when your aim in performing the experiment and communicating results is to comment on values outside your data range. If you are only wishing to discuss results within your data range, do not extrapolate.

The dashed purple lines show interpolation – it is possible to read off an expected value for the rate at a substrate concentration not actually tested (in the example shown we can see that a substrate concentration of 3.3 a.u. would be expected to give a rate of 1.8 a.u.).

The dashed orange line shows extrapolation – a substrate concentration of 10.0 a.u. is higher than the highest concentration tested but we can still suggest a possib corresponding rate of 6.0 a.u.

Tips for plotting line graphs

Plotting a graph is fairly straightforward but there are a few tips which may help you:

Be very careful plotting points accurately

Use appropriate linear scales on axes

Use 'sensible' scales, for example using a decimal or straightforward scale

Scales must be chosen so that all points fall within the graph area

Axes must be labelled, with units included

Make the graphs as large as the available space allows

Use an informative title.

A line (or curve) of best-fit should be drawn to identify trends. The line must be smooth and have a balance of data points above and below the line.

Sometimes extrapolation of data is required, for example to determine the intercept with the *y*-axis. To do this you need to extend the line of best fit to the appropriate point. Extrapolation can be used to predict values beyond the existing data set based on the current trend.

In some situations the line of best fit needs to be drawn through the origin, for example for rate–concentration graphs. However, the line of best fit should only go through the origin if the data and trend allow it.

Scattergrams

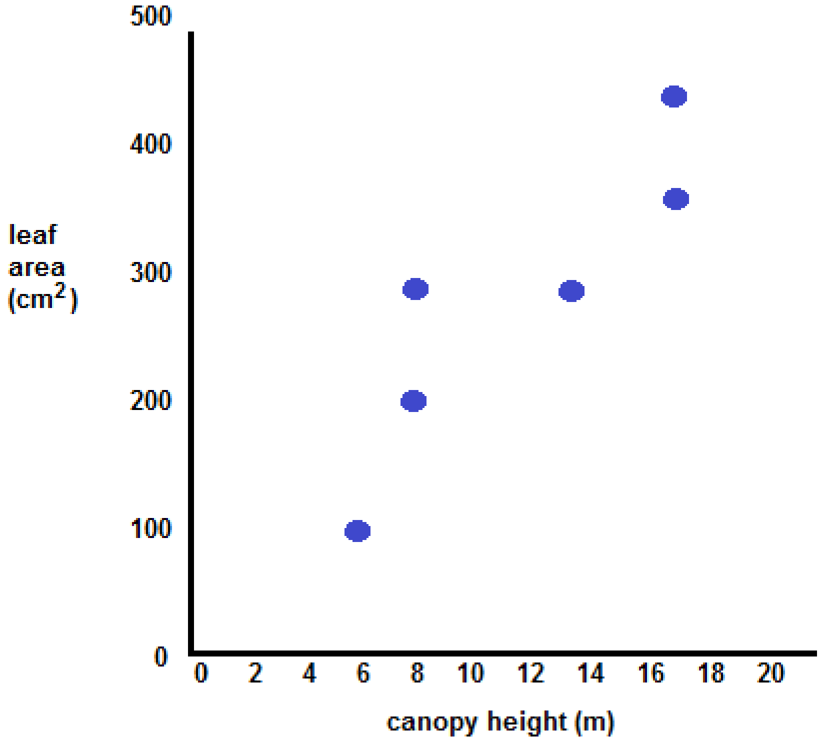
In some cases data which can look superficially very similar to the line graph scenario is better presented as a scattergram.

Once again we have two variables for each data point. It might also be the case that we have a variable we can identify as the independent and a variable we would call the dependent. But scattergrams are also appropriate when this is not the case. We might be interested in a possible association between two variables without being in a situation to change one variable and see what effect it has on the other variable.

The clearest case where we would choose to present data as a scattergram rather than as a line graph is when we have obtained our data set by sampling a natural or pre-existing situation rather than by performing a lab-based experiment where we control most variables, change one and measure another.

For example we could sample tree species in a woodland. For each species we could measure two variables (height of canopy and surface area of leaf).

The resulting data could be appropriately presented as a scattergram. In this case we might choose to add a line very much like a line of best fit but in this case it is intended to highlight the suspected association between the variables. A quantitative analysis of the suspected association could be done using the statistical test 'Spearman's rank correlation' – see M1.9.



M3.2 – Plot two variables from experimental or other data

Quiz

A calibration curve

Calibration curves and lines are used to plot the relationship between two variables so that the resulting curve or line can be used to read off the value of an unknown sample. They are examples of line graphs.

For example, in colorimetry a calibration curve is used to establish the relationship between the absorption of a particular wavelength of light by a solution, and the concentration of a coloured solution.

This technique may be used to plot the data for the absorption of a reducing sugar of known concentrations tested with Benedict's reagent, for example:

Reducing sugar concentration (mol dm^{-3})	0.0	0.2	0.4	0.6	0.8	1.0
Absorbance (AU)	0.95	0.83	0.71	0.62	0.50	0.41

a) Plot the data to produce a calibration graph.

b) If a reducing sugar solution of unknown concentration is now tested with Benedict's reagent, the absorbance value allows us to read off the concentration of the unknown solution.

Solution A gives an absorbance reading of 0.56 arbitrary units.

What is the concentration of reducing sugar?

M3.3 – Understanding that *y* = *mx* +*c* represents a linear relationship

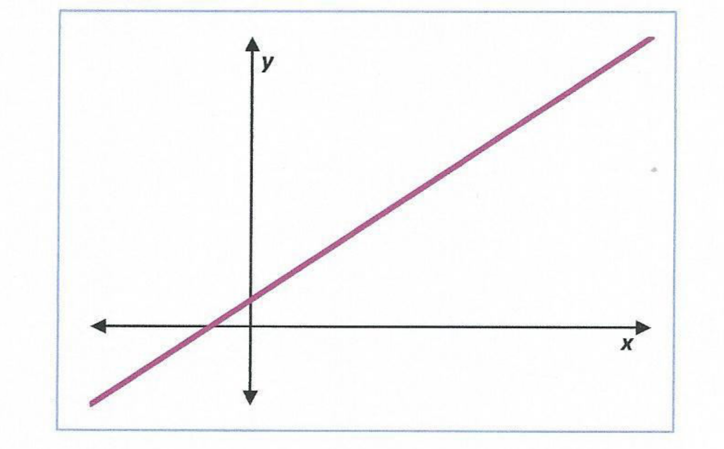
Tutorials

Learners may be tested on their ability to:

- predict/sketch the shape of a graph with a linear relationship, e.g. the effect of substrate concentration on the rate of an enzyme-controlled reaction with excess enzyme.

Linear relationships represented by *y* = *mx* + *c*

As we discussed in section M3.1, you should be able to identify a linear relationship when given a graph that looks like this. You also must be able to sketch a graph when given a linear relationship.



A sloping straight line shows that the dependent variable on the y axis is proportional to the independent variable on the x axis. To demonstrate something is proportional to something else we use the symbol α.

Dependent variable (y axis)

Independent variable (x axis)

y α x

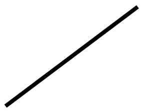
Mathematically this is represented by the equation $y = mx + c$. The letter “m” is the gradient of the line – we explain how to calculate this in section M3.5, and “c” is the value of the intercept on the y axis, which we explain in section M3.4.

m → Gradient of the line

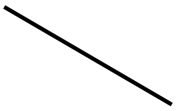
c → Y intercept

You need to be able to determine whether the linear relationship is positive or negative. If the line slopes up from left to right this shows a positive relationship, and the gradient “m” will be a positive number. If the line slopes down from left to right, it’s a negative relationship and the gradient “m” is a negative number.

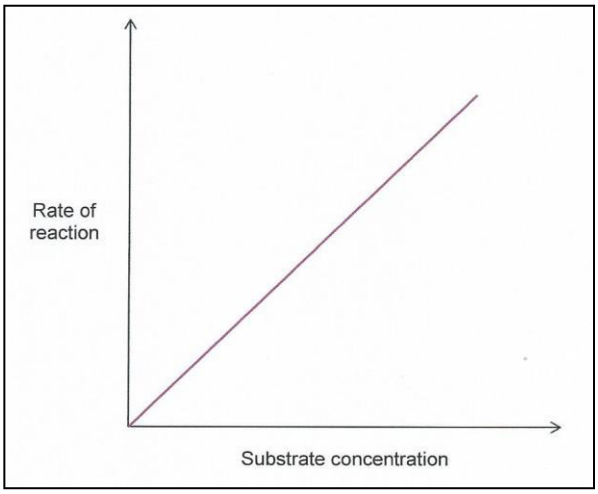
Positive relationship – positive gradient (m)



Negative relationship – negative gradient (m)



For example, this graph of rate of an enzyme reaction against substrate concentration shows a positive relationship.



In this example the intercept on the y axis (represented by “c” in the equation) is zero. Therefore in this case the equation becomes y = mx + 0, or y = mx.

$$y = mx + 0$$

$$y = mx$$

M3.3 – Understand that $y = mx + c$ represents a linear relationship

Quiz

1. Sketch a graph to show how cardiac output changes as heart rate increases if stroke volume does not change.

   Note: if you have covered topic **3.1.2 Transport in animals** you should already know the equation that relates cardiac output to stroke volume and heart rate. In an exam you could be tested, as part of a question like this, on whether you can remember the equation but here is a reminder:

$$cardiac\ output = heart\ rate\ \times stroke\ volume$$

2. Data from an experiment show that the mean height attained by seedling shoots in the first 48 hours after germination is directly proportional to the concentration of auxin applied to the germinating seed. Seedlings without any auxin treatment grew to a mean height of 12 mm. Seedlings exposed to the maximum concentration of auxin grew to a mean height of 31 mm.

   Sketch a graph to show the relationship between mean seedling height and auxin concentration.

M3.4 – Determine the intercept of a graph

Tutorials

Learners may be tested on their ability to:

- read off an intercept point from a graph, e.g. compensation point in plants.

Determining the intercept of a graph

An intercept is where one line on your graph crosses another. This could be where a line of best fit crosses either the x or y axis. Or it could be where two lines of best fit cross each other.

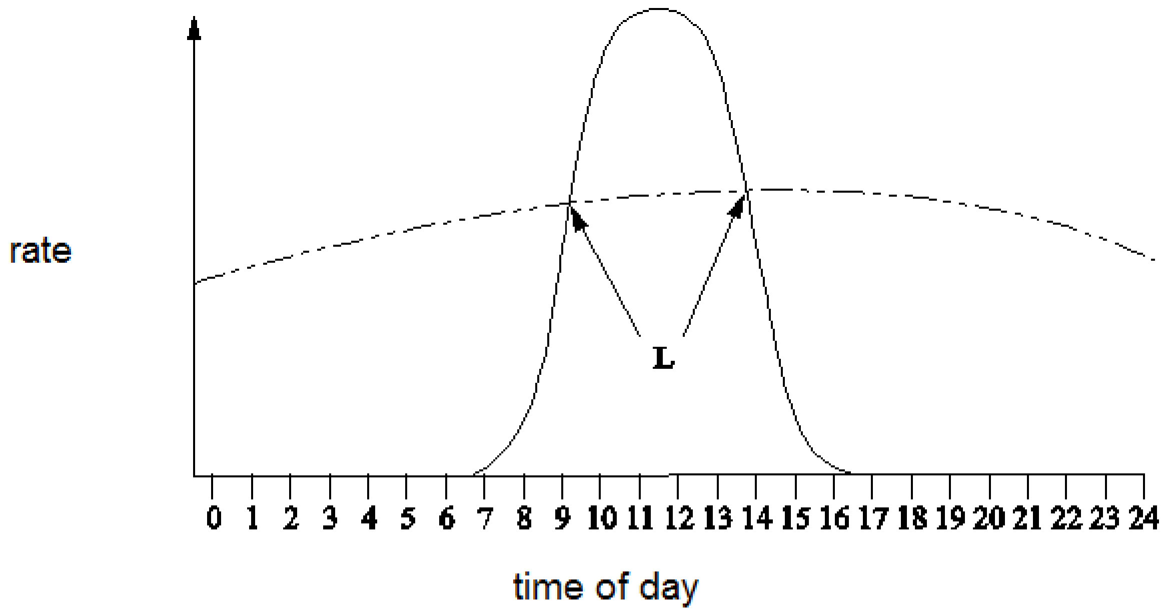
In section M3.3 we talked about graphs represented by the linear relationship $y = mx + c$, where "m" represents the gradient of the line, and "c" represents the y-intercept. Finding the y-intercept is easy – it is simply where the line crosses the y axis, where $x = 0$. Similarly, the x-intercept is where the line crosses the x axis, where $y = 0$.

Intercepts = where the line crosses either the x or y axis

Note: if your x axis doesn't start from zero, you will not be able to read off the y intercept in this way. If you need to find the y intercept you should start your x axis from zero. Likewise, if you are aiming to find the x intercept when you have plotted your data you should make sure that your y axis starts from zero.

As well as finding the intercepts of a graph, you also need to be able to find and read off the intersection of two or more data series from a graph. This is basically where two different curves on the same axes intersect.

For example, you may be asked to find the compensation points in plants. Compensation points are where the rate of photosynthesis exactly matches the rate of respiration. In this graph the compensation points (L) are where the graphs representing the rate of photosynthesis and the rate of plant respiration intersect.



Key

——————— rate of photosynthesis

— - - — - - — - - rate of plant respiration

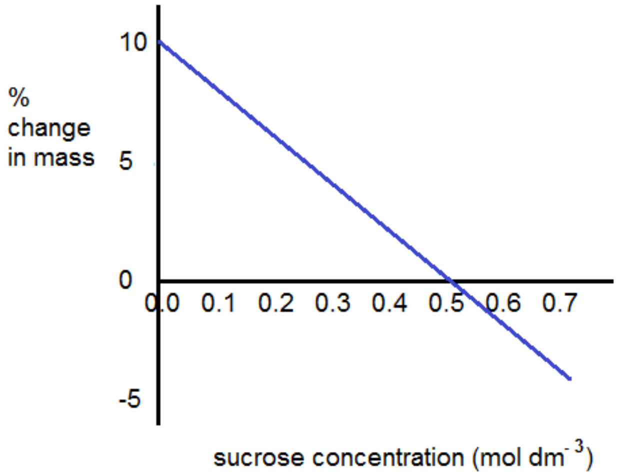
M3.4 – Determine the intercept of a graph

Quiz

1. An experiment was carried out to find the water potential of the cells in potato tubers.

   Cylinders of potato were cut from potatoes and weighed. These cylinders were then immersed in sucrose solutions of different concentrations for 4 hours. The cylinders were then weighed again and the percentage change in mass was recorded.

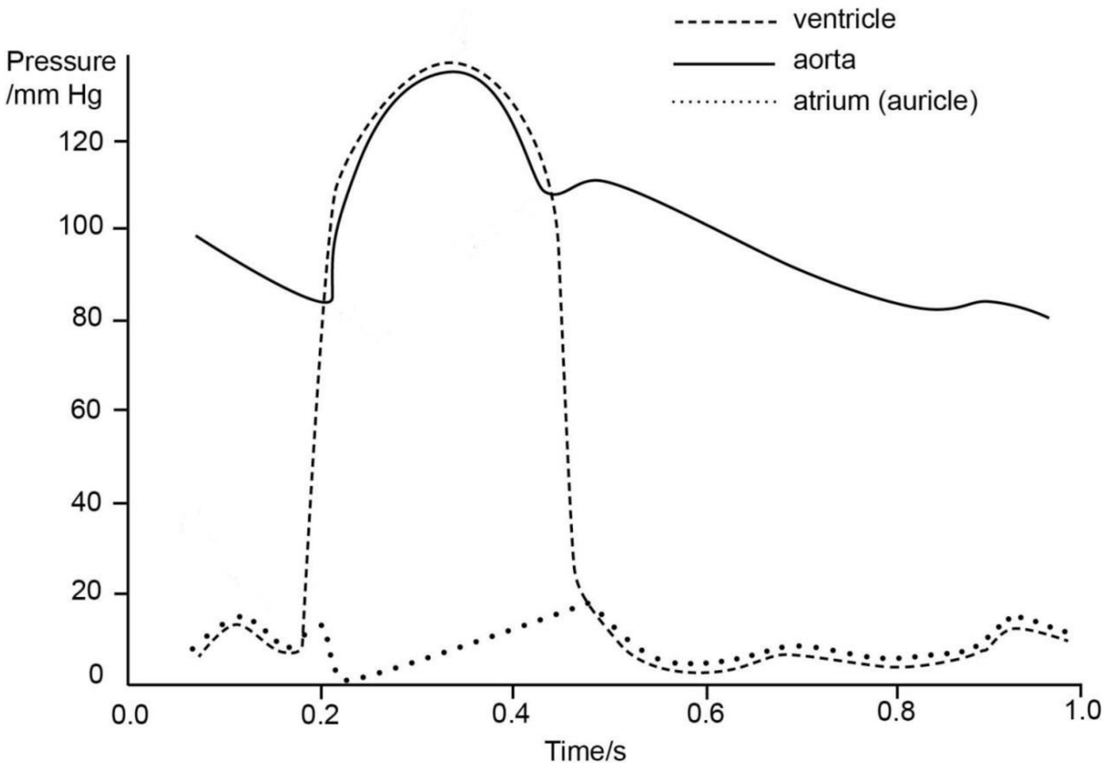
   The results were plotted and the line of best fit is shown in the graph below.



What are the intercepts in this graph?

Which intercept will be used to find the water potential of the cells?

2. Measurements were made of pressure changes within the left side of the heart and the aorta during the cardiac cycle of a healthy adult human. The intercepts of the various curves indicate moments in the cycle when pressure is equal in the two chambers or vessels. This is when valves open and close.



Identify the intercepts and give their meanings in terms of valve opening/closing.

M3.5 – Calculate rate of change from a graph showing a linear relationship

Tutorials

Learners may be tested on their ability to:

- calculate a rate from a graph, e.g. rate of transpiration.

Calculating rate of change from a graph

As we explained in section M3.3, a graph with a linear relationship can be represented by the formula $y = mx + c$. The “c” represents the y-intercept, as we demonstrated in section M3.4. In this section we’re going to explain how to work out the gradient of the line, which is represented by the letter “m”.

y = mx + c

“c” → y-intercept

“m” → gradient

To calculate the gradient, we can use the formula:

gradient equals the change in y divided by the change in x:

$$Gradient = \frac{'change\ in\ y'}{'change\ in\ x'}$$

To use this formula you take two points on the line of the graph. Measuring the vertical distance between the points gives the change in y, and measuring the horizontal distance between the points gives the change in x. Dividing the change in y by the change in x gives the gradient of the line.

The gradient of a linear graph tells us the rate of change of one variable with respect to another. In other words you could say that the gradient expresses how quickly *y* changes as *x* changes.

Rate of change

How quickly *y* changes as *x* changes

What does this mean in a biological context?

In many cases our independent variable (x) is time. This fits naturally with describing the gradient simply as ‘rate’.

For example if we are studying transpiration we can take readings of the total volume of water lost (our dependent or ‘y’ variable) as time passes (our independent or ‘x’ variable).

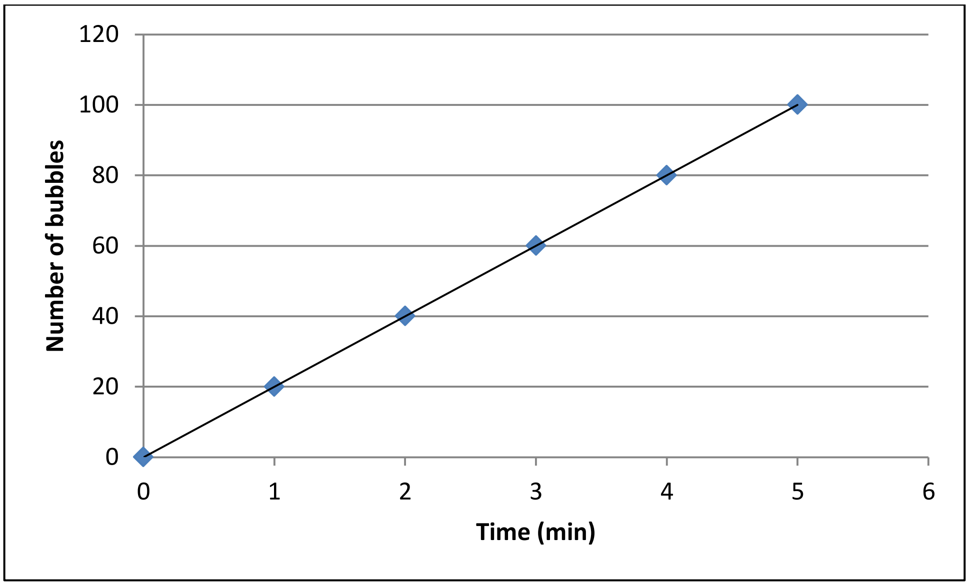
When we plot these results as a line graph the gradient of the line gives us the rate of transpiration.

M3.5 – Calculate rate of change from a graph showing a linear relationship

Quiz

The rate of photosynthesis in a piece of pondweed was measured by the number of bubbles of oxygen released over time.

The results were plotted as a line graph:



Find the gradient (the rate of photosynthesis).

M3.6 – Draw and use the slope of a tangent to a curve as a measure of rate of change

Tutorials

Learners may be tested on their ability to:

- use this method to measure the gradient of a point on a curve, e.g. quantity of product formed plotted against time when the concentration of enzyme is fixed.

Using tangents to measure rate of change

In section M3.5 we explained how to calculate the gradient of a straight line to work out the rate of change. In a straight line graph the gradient is constant throughout. But if we have a curved line then the gradient is different at different points on the curve.

So how do we work out the rate of change at a point on a curved graph?

An easy way is to draw a tangent to the curve. A tangent is a straight line that is drawn so it just touches the curve at a singular point. The slope of this line matches the slope of the curve at just that point. You then simply find the gradient of the line, as described in M3.5, to find the gradient at that point on the curve.

This gives you the rate of change at a particular point on a curve.

There are a few things to remember when drawing tangents: Firstly, always use a ruler and a pencil. You need to make sure the line you draw is dead straight, and using a pencil is essential in case you make a mistake. Choose the point where the tangent is to be taken and line the ruler up to that point. Make sure none of the line of the curve is covered by the ruler; the curve needs to be entirely visible whilst the tangent is drawn.

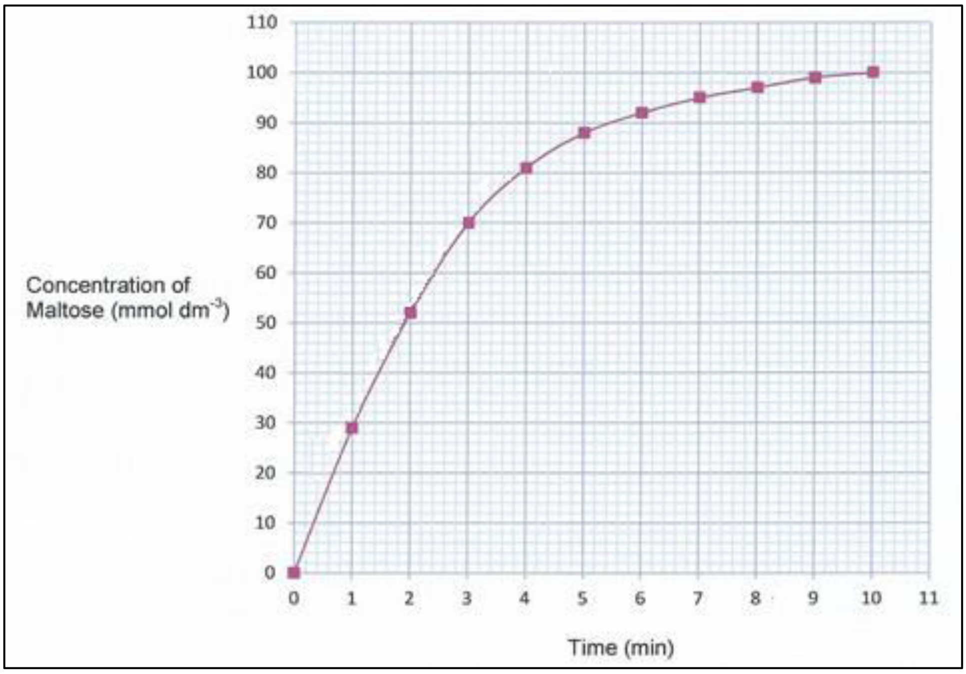
1) Use a ruler and a pencil
2) Line the ruler up to the point where the tangent is to be taken
3) Make sure none of the curve is covered by the ruler

Once you have drawn the tangent to a curve you can then work out the gradient of the tangent in the same way as we explained in section M3.5. This will give you the rate of change of the curve at that particular point.

M3.6 – Draw and use the slope of a tangent to a curve as a measure of rate of change

Quiz

This graph shows concentration of maltose produced over time for an enzyme-controlled reaction. Find the rate of maltose production at 2 min.



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