Maths tutorial booklet for

M2: Algebra

Name:_____

Target grade:_____

Quiz scores:

M2.1 Understand and use symbols =, ‹, ‹‹, ››, ›, α, ~   =

M2.2 = Change the subject of an equation

M2.3   Substitute numerical values into algebraic equations using appropriate units for physical quantities **and** M2.4 Solve algebraic equations =

M2.5 Using logarithms =

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Maths skills – M2.1 Understand and use symbols =, ‹, ‹‹, ››, ›, α, ~

Tutorials

Learners may be tested on their ability to:

- use these symbols appropriately and correctly in their given contexts

- understand these symbols in the contexts of formulae given.

Understanding symbols

There are many mathematical symbols that you need to understand and be able to use in lots of different biological contexts. You will know that this symbol (=) means 'equal to' and if you see this symbol (~), it means approximately equal to.

This symbol (<) means 'less than' and is used to show that the value on the left side of the symbol is smaller than the value on the right side of the symbol. Likewise, its opposite is (>), which means greater than. Basically the open side of each indicates the bigger number and the pointed side indicates the smaller number. If a value is much less than or much greater than another, then you use these symbols: (<<), (>>).

=	Equals
~	Approximately equal to
<	Less than
>	Greater than
<<	Much less than
>>	Much greater than
α	Proportional to

Finally, this symbol ($\propto$) means "proportional to". If two quantities *A* and *B* are *directly proportional* then the appropriate mathematical statement is $A \propto B$

When A gets bigger, so does B.

$$A \propto B$$

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If the two quantities are *inversely proportional* then the appropriate relationship between *A* and *B* is $A \propto \dfrac{1}{B}$

A is proportional to one over B. When A gets bigger, B gets smaller.

$$A \propto \frac{1}{B}$$

Using symbols

You might report that use of fertilizers on agricultural land was found to be proportional to algal growth in surrounding water bodies, or, fertilizer use was proportional to algal growth.

Use of fertilizers $\propto$ Algal growth

In the same study you might find that fertilizer use, F, on the agricultural land was inversely proportional to available oxygen, O, in surrounding waters, or fertilizer use was inversely proportional to oxygen availability.

$$F \propto \frac{1}{O}$$

Use of nitrogen fertilizer, NF, was greater than the use of organic fertilizer, OF.

NF > OF

But the growth of algae next to land using nitrogen fertilizer G_n was approximately equal to growth next to land with organic fertilizer, G_o.

$G_n \sim G_o$

These symbols are important when reporting relationships between two or more parts of a study. In biology though, information like this would be supported with data and specific relationships would be reported, but for general accounts of a biological scenario, these symbols might be used. You need to be able to recognise and understand all of these symbols when they are used in biological contexts.

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Maths skills – M2.1 Understand and use symbols =, ‹, ‹‹, ››, ›, α, ~

Quiz

1. What do the following symbols mean?

=	
~	
<	
>	
<<	
>>	
α	

2. Use the most appropriate symbol to complete the following statements:

   a) 10 _____ 20

   b) 1,000,000 _____ 2

   c) 10.45 _____ 10.44

   d) 12 _____ 12

3. Use symbols to express these biological statements more briefly:

   a) The number of wolves is less than the number of red deer.

   b) There are approximately as many females as males in the population.

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c) The number of synapses in the human brain is much greater than the number of neurones

d) The above ground biomass of plant species in the meadow in June is inversely proportional to the number of sheep grazing the meadow in May

Maths skills – M2.2 Change the subject of an equation

Tutorials

Learners may be tested on their ability to:
- use and manipulate equations, e.g. magnification

Changing the subject of an equation

There are many important equations or formulae that are used in many biological contexts. It is essential that you are able to rearrange these formulae to make the "unknown" part of the equation the subject.

The golden rule with equations is to make sure that one side always equals the other. The equals sign is the crucial part, so if you do something to one side of the equation, you must do the same to the other side.

The part of the equation that you are trying to find, the 'unknown', always goes on the left hand side, but the equation can be written either way round, so a=bc is exactly the same as bc=a, but if it is a that is the subject of the equation, it goes on the left hand side.

For example three quantities *a*, *b* and *c* are linked by the simple relationship *a* equals *b* times *c*.

$$a = bc$$

To make *b* the subject of the equation you need to get it on its own. So we need to divide both sides of the equation by *c*. *c* then cancels out on the right hand side, and leaves the formula *a* divided by *c* equals *b*.

$$\frac{a}{c} = b$$

The new subject of the equation is *b*, so it must go on the left hand side, so the equation is written the other way around:

$$b = \frac{a}{c}$$

For example, as explained in section M1.8, a common equation used in biological drawing links magnification, image size and object size. The equation for this is magnification = the size of the image divided by the actual size of the object.

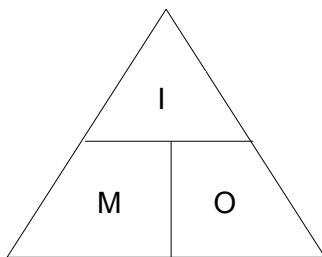$$magnification = \frac{size\ of\ image}{size\ of\ real\ object}$$

If you know the magnification, and the size of the image, you can work out the actual size of the object. So you need to rearrange the equation to make the actual size of the object the subject. Your first problem is that what you want as the subject is a denominator in a fraction. So you would start by multiplying both sides by the actual size of the object. This allows you to cancel this term from the right hand side of the equation. So you now have magnification times the actual size of the object equals the size of the image.

$$magnification\ x\ size\ of\ real\ object = size\ of\ image$$

You now divide both sides of the equation by the magnification, which lets you cancel magnification from the left hand side, leaving the actual size on its own, and has size of image divided by magnification on the right hand side.

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$$size\ of\ real\ object = \frac{size\ of\ image}{magnification}$$

Many equations can be represented as a formula triangle. For example, the magnification formula in a triangle looks like this, where 'I' represents the size of image, 'm' is the magnification and 'o' is the size of the real object.

Although these are very useful, it is essential that you understand how to rearrange formulae yourself, and fully understand the equals sign. If you rely on the triangle, when you come to rearrange, for example, the equation for a straight line:

$$y = mx + c$$

or Pythagoras's equation:

$$a^2 + b^2 = c^2$$

you will not be able to do it.

Maths skills – M2.2 Change the subject of an equation

Quiz

1. Rearrange the equation

$$speed = \frac{distance}{time}$$

so that

a) Distance is the subject of the equation

b) Time is the subject of the equation

2. Rearrange the equation

$$pressure = \frac{force}{area}$$

so that

a) Force is the subject of the equation

b) Area is the subject of the equation

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3. Rearrange $y = mx + c$ to make c the subject of the equation

4. Rearrange $a^2 = b^2 + c^2$ to make b^2 the subject of the equation

Maths skills – M2.3 Substitute numerical values into algebraic equations using appropriate units for physical quantities

Maths skills – M2.4 Solve algebraic equations

Tutorials

Learners may be tested on their ability to:

- Use a given equation   e.g. Simpson's index of diversity

$$D = 1 - \sum \left(\frac{n}{N}\right)^2$$

- Solve equations in a biological context

    e.g: cardiac output = stroke volume x heart rate

Substituting numerical values into algebraic equations

Now that you understand how to rearrange formulae (M2.2 Change the subject of an equation) you need to be able to solve them. The first step of this process involves putting numerical values that are either given in the question or that you have already worked out into the equation. You then need to work out what the equation is now with the new numbers in it. For example, if you have the equation $y = 2x + 5$ and were told in the question that $x = 4$ then the equation for y would be

y = 2 multiplied by 4, plus 5.

Two times 4 is 8

giving $y = 8 + 5$. So y=13

$$y = 2x + 5 \qquad x = 4$$

$$y = 2(4) + 5$$

$$y = 8 + 5$$

$$y = 13$$

There are a few rules that you need to remember. First is when you are substituting a negative number into x^2. Brackets are essential for this sort of question. For example, substituting in $x = -2$ gives us (-2)². This means -2 times -2. Without the brackets you may mistake the question for 2² with a minus in front of the answer.

$$x^2 \qquad x = -2$$

$$x^2 \to (-2)^2 \qquad ☺$$

$$x^2 \to -2^2 \qquad X$$

Another important fact to remember is that when you multiply (or divide) two negative numbers, the answer is a positive number.

negative x negative = positive

Therefore, this means when you substitute in $x = -2$ into x^2 to get -2 times -2, the answer is plus 4.

$$x^2 \to (-2)^2$$

$$(-2)^2 = -2 \times -2 = 4$$

Remember, it is always true that two negative numbers multiplied or divided equal a positive:

$-3 \times -5 = +15$

Whilst if only one of them is negative then the answer is negative:

$-3 \times 5 = -15$

Addition and subtraction are different. Think of the number line for these calculations.

For example:

$-3 + -5$

Here you have minus three and then go a further minus five down the number line. If you take away -5 from -3 you are going the opposite direction, so you are adding 5 to minus three, which gives you plus 2.

−3 + −5

-3 − 5

-3 − 5 = -8

−3 − −5

-3 + 5

-3 + 5 = 2

An example of where understanding how to substitute numerical values into an equation is important in biology is when using Simpson's index of diversity. This gives an index of diversity by taking into account the total number of organisms (N) and the number of each particular species (n).

Simpson's Index of Diversity states:

$$D = 1 - \sum \left(\frac{n}{N}\right)^2$$

N = Total number of organisms   n = number of a particular species

For example, here we have the numbers of 3 different species of trees in two samples.

	Number of individuals	
Tree species	**Sample 1**	**Sample 2**
Elm	250	15
Willow	385	61
Oak	365	924
Total	1000	1000

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For each species, we need to work out the number of that particular tree species (*n*) divided by the total number of trees (N), and square the answer. Then we need to add the answers for all of the tree species together. So, for this example we have 250 elm trees, 385 willow trees and 365 oak trees – these are all the *n* values. The total number of organisms is 1000 so each of these species numbers needs to be divided by 1000. We then square each term, using brackets to help separate them, and finally add them all together. This gives an answer of 0.34395.

$$\left(\frac{250}{1000}\right)^2 + \left(\frac{385}{1000}\right)^2 + \left(\frac{365}{1000}\right)^2 = 0.34395$$

We have worked out that $\sum\left(\frac{n}{N}\right)^2$ = 0.32395. So $D = 1 - 0.34395$. This gives an answer of 0.65605.

$$D = 1 - 0.34395 = 0.65605$$

For sample 2 we can go through the same process. The index of diversity is 0.142278

$$\left(\frac{15}{1000}\right)^2 + \left(\frac{61}{1000}\right)^2 + \left(\frac{924}{1000}\right)^2 = 0.857722$$

$$D = 1 - 0.857722 = 0.142278$$

Comparing the diversity values for samples 1 and 2 shows that sample 1 has a higher value for D and therefore a higher diversity according to the Simpson's index of diversity.

Higher value for *D* = a higher diversity

Solving algebraic equations

Solving an equation involves substituting values into a formula to calculate an unknown. This may involve rearranging the equation so that the unknown is on one side and the other values on the other side. Sections M2.2 and M2.3 explain how to rearrange equations and substitute in numerical values into a formula. You will then be able to solve your equation.

1) Rearrange the equation
2) Substitute values
3) Solve

For example, cardiac output is the volume of blood pumped by the left ventricle of the heart in one minute. This is calculated by multiplying the volume of blood pumped in one beat (stroke volume) by the number of beats in a minute (heart rate).

cardiac output = stroke volume x heart rate

By rearranging the equation and substituting in what we know, we can find the unknown.

Take this question: Heart rate during exercise is 185 bpm. The cardiac output is 10 dm^3 min^{-1}. What is the stroke volume?

heart rate = 185 bpm

cardiac output = 10 dm^3 min^{-1}

stroke volume = ?

First of all we need to rearrange the equation to make the stroke volume the subject of the equation. By looking at our original equation we can see that to make the stoke volume the subject of the equation we need to divide both sides by the heart rate. This gives is the equation: stroke volume = cardiac output/heart rate

cardiac output = stroke volume x heart rate

stroke volume = cardiac output/heart rate

Now we have our equation we need to substitute in the values we know. We know that the heart rate is 185 bpm and the cardiac output is 10 dm^3 min^{-1}. Therefore we now have the equation: stroke volume = 10/185

stroke volume = 10/185

By solving this equation we get a stroke volume of 0.054 dm^3. This is better expressed as 54 cm^3.

stroke volume = 0.054 dm^3 = 54 cm^3.

Maths skills – M2.3 Substitute numerical values into algebraic equations using appropriate units for physical quantities

Maths skills – M2.4 Solve algebraic equations

Quiz

1. Use the Simpson's index of diversity for the following questions to work out which sample has the higher diversity.

a) Using quadrats, samples were taken from two fields giving the following results.

	Number of individuals	
Plant species	**Field A**	**Field B**
Daisy	300	20
Buttercup	335	49
Dandelion	365	931
Total	1000	1000

Calculate Simpson's index of diversity (D) for each field using the formula:

$$D = 1 - \Sigma \left(\frac{n}{N}\right)^2$$

Field A

Field B

Which field **A** or **B** has higher diversity?

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b) Two areas of embryo dunes were sampled giving the following results.

	Number of individuals	
Plant species	**Area A**	**Area B**
Saltwort	40	20
Sandwort	55	15
Frosted orache	30	75
Sea rocket	90	80
Total	**215**	**190**

Calculate Simpson's index of diversity (D) for each area using the formula:

$$D = 1 - \Sigma \left(\frac{n}{N}\right)^2$$

Area A

Area B

Which area **A** or **B** has higher diversity?

2. Ventilation rate is the volume of air exchanged in one minute. Breathing rate is the number of breaths per minute. Tidal volume is the volume of air exchanged in one breath.

This gives the equation: $ventilation\ rate = tidal\ volume \times breathing\ rate$

If tidal volume is 0.5 dm^3 and ventilation rate is 6 dm^3 min^{-1}, what is the breathing rate?

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3. If the mean number of daisies in a quadrat 0.5 m x 0.5 m is 10, what will be the total number of daisies in a field measuring 100 m x 50 m?

4. When water flows along a tube, such as a blood capillary, the flow of liquid can either be laminar or turbulent.

   This can be calculated by putting numbers into the following equation.

   If the result for N_R is greater than 3000 flow is turbulent, and less than 2000 flow is laminar.

   The equation is:

   $$N_R = \frac{\rho v D}{\mu}$$

   Where
   ρ = the density of the fluid   (kg m^{-3})
   v = flow velocity   (m s^{-1})
   D= the diameter of the tube   (m)
   μ = the viscosity   (kg m^{-1}s^{-1})

   Calculate N_R for the following:

   a) The velocity through an aorta of diameter 2 x 10^{-2} m is 0.3 m s^{-1}.

   The density of blood is 10^3 kg m^{-3}.

   The viscosity of blood is 4 x 10^{-3} kg m^{-1}s^{-1}

$$N_R = \frac{10^3 \times 0.3 \times 2 \times 10^{-2}}{4 \times 10^{-3}}$$

$N_R =$

Is the flow laminar or turbulent?

b) Flow of water through a tube of diameter 4 x 10^{-2} m, supplying a fish tank, was found to be
0.5 m s^{-1}.

The density of water is 10^3 kg m^{-3}.

The viscosity of water is 10^{-3} kg m^{-1} s^{-1}.

$$N_R = \frac{10^3 \times 0.5 \times 4 \times 10^{-2}}{10^{-3}}$$

$N_R =$

Is the flow laminar or turbulent?

Maths skills – M2.5 Using logarithms

Tutorials

Learners may be tested on their ability to:

- Use a logarithmic scale in the context of microbiology

  e.g. growth rate of a microorganism such as yeast

Logarithms

Logarithms are powers to which a base is raised. For example, if we take the following expression:

'10 to the power 2'　　　　　(written as 10^2)

10 is the base,
2 is the power.

10^2 = 100

Since 10 to the power 2 equals 100, log of 100 equals 2 because the log of 100 equals the power to which the base of 10 is raised.

Log to the base 10 is written log with a subscript 10 ($\log_{10}$), but in general the term log is taken to mean log to the base 10.

the power of 10 that gives 100 is 2

log 100 = 2

Logarithms are really useful for providing a better scale when dealing with quantities that vary considerably.

For example, imagine sketching a graph where the scale goes from zero to 100,000, but with values such as 10 and 50 that are of interest.

If you were to plot this on a normal scale, lower values would all clump together. If however, you plotted the log of each value, the points on the graph would be much easier to separate. If, on this scale of zero to 100,000 you had values to plot at points 10, 50 100, 3,000 and 90,000, you could plot the value that was equal to these as powers to which 10 is raised.

So Log 10 is equal to 1 because the power that base 10 has to be raised to give a value of 10, is 1.

Log 50 is 1.7. This is because 10 raised to the power 1.7 equals 50.

Log 100 is 2,

and Log 3000 equals 3.48, because 10 to the power 3.48 equals 3000.

Finally log 90,000 equals 4.95.   So on your graph, instead of plotting 10, 50, 100, 3000 and 90,000, you would plot 1, 1.7, 2, 3.48 and 4.95. You can see that these are much more sensible numbers to use to make visual comparisons.

Logarithm scales are used when measuring sizes of earthquake – the Richter scale. So an earthquake that measures 6 has a value of 10 to the power 6, and is much smaller than an earthquake that measures 7 because this one is 10 to the power 7, a factor of 10 times larger. It is the log of the value that is recorded.
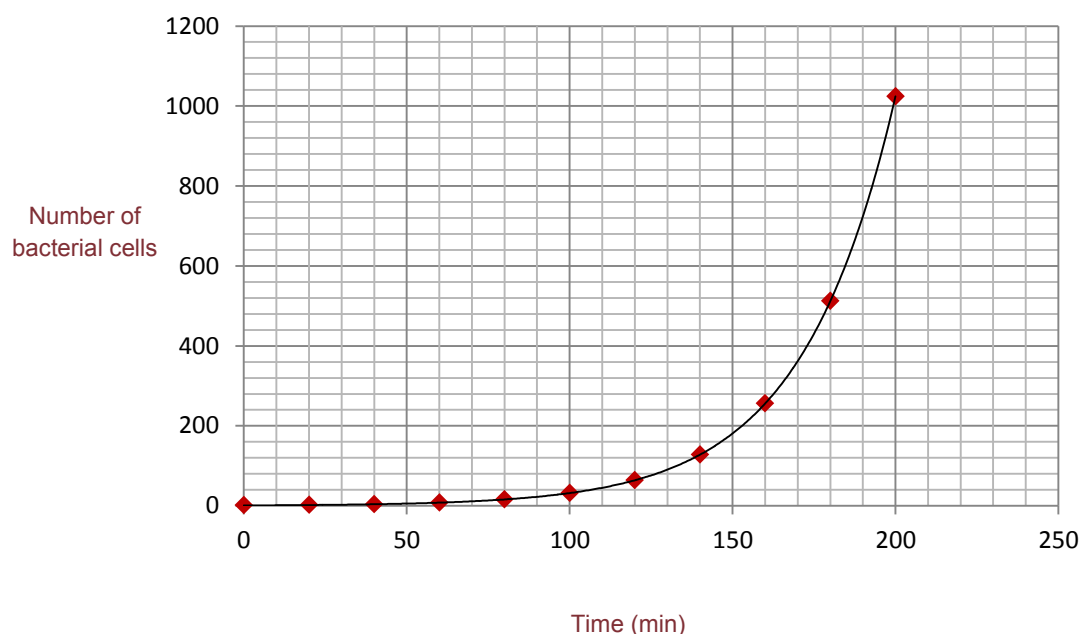
Mostly we use log to the base 10, written as log. But occasionally we use log to the base 2.7182818…. the number goes on for ever… this is the number *e*. You will see it written as *l*n, which is simply another way of writing log to the base *e*. *e* is the exponential function, and is used, for example, when looking at the growth of populations.

Logarithms are very useful in microbiology when studying the growth rates of microorganisms. As discussed in section M0.5, bacterial cells multiply exponentially with one
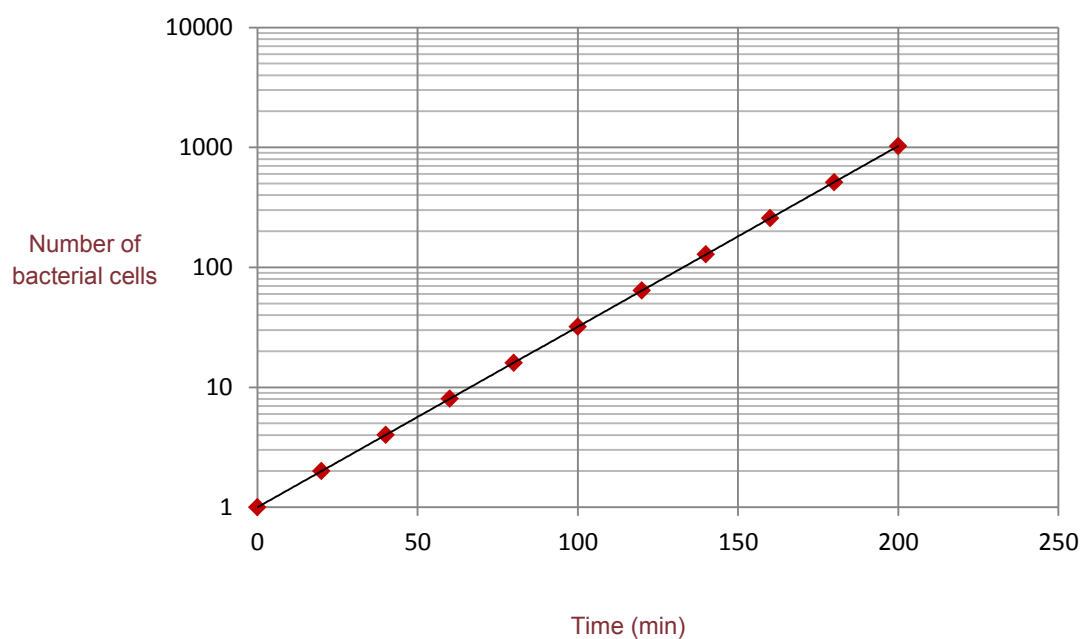
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bacterial cell usually dividing every 20 minutes or so under standard conditions. The cells produced will form a sequence of numbers:

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 …

If you plot these data in the standard way you get an exponential growth curve that looks like this:

However, if instead you plot the data using a logarithmic scale (or take the log of each number and plot these values) you get a straight line relationship like this, because the logarithm is the power to which a base is raised, and if you plot the log you are taking a power relationship and plotting it as a linear relationship.

Data plotted using a logarithmic scale can be more manageable when visualising and interpreting the exponential relationship.

Maths skills – M2.5 Using logarithms

Quiz

1. You're doing an experiment where a bacterium divides every 30 minutes under standard conditions. There were 10 bacteria in your starting culture. You want to plot the bacterial growth rate over a period of 4 hours and take measurements every half an hour.

   Work out the sequence of numbers, complete the table below and plot two graphs, one of which should be a logarithmic graph.

Time (min)	Number of bacteria	Log of the number of bacteria
0	10	

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2. To represent the following animals on a bar chart and their densities in a woodland, you would need to take the log of the number of animals in each category.

Complete the table for the following:

Animal	Number of animals in 100 m^2	Log of the number of animals
Earthworm	50,000	
Beetle	2,000	
Mouse	750	
Fox	12	
Deer	3	

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